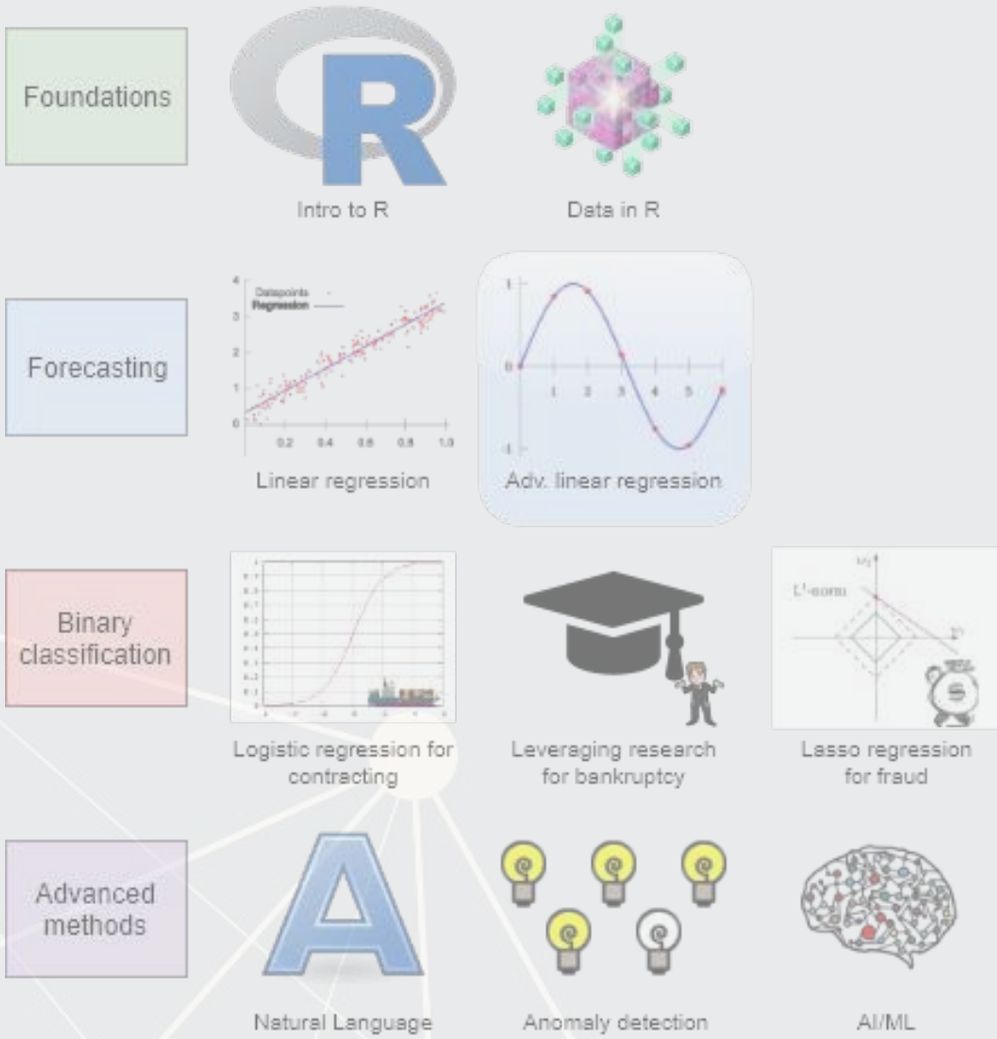
# ACCT 420: Advanced linear regression

## Session 4

Dr. Richard M. Crowley

# Front matter

# Learning objectives




- **Theory:**
  - Further understand:
    - Statistics
    - Causation
    - Data
    - Time
- **Application:**
  - Predicting revenue quarterly and weekly
- **Methodology:**
  - Univariate
  - Linear regression (OLS)
  - Visualization

# Datacamp

- Explore on your own
- No specific required class this week

# Based on your feedback…

- To help with replicating slides, each week I will release:
  1. A code file that can directly replicate everything in the slides
  2. The data files used, where allowable.
     - I may occasionally use proprietary data that I cannot distribute as is – those will not be distributed
- To help with coding
  1. I have released a practice on mutate and ggplot
  2. We will go back to having in class R practices when new concepts are included
- To help with statistics
  1. We will go over some statistics foundations today

# Assignments for this course

- Based on feedback received today, I may host extra office hours on Wednesday

> Quick survey: rmc.link/420hw1

# Statistics Foundations

# Frequentist statistics

> A specific test is one of an infinite number of replications

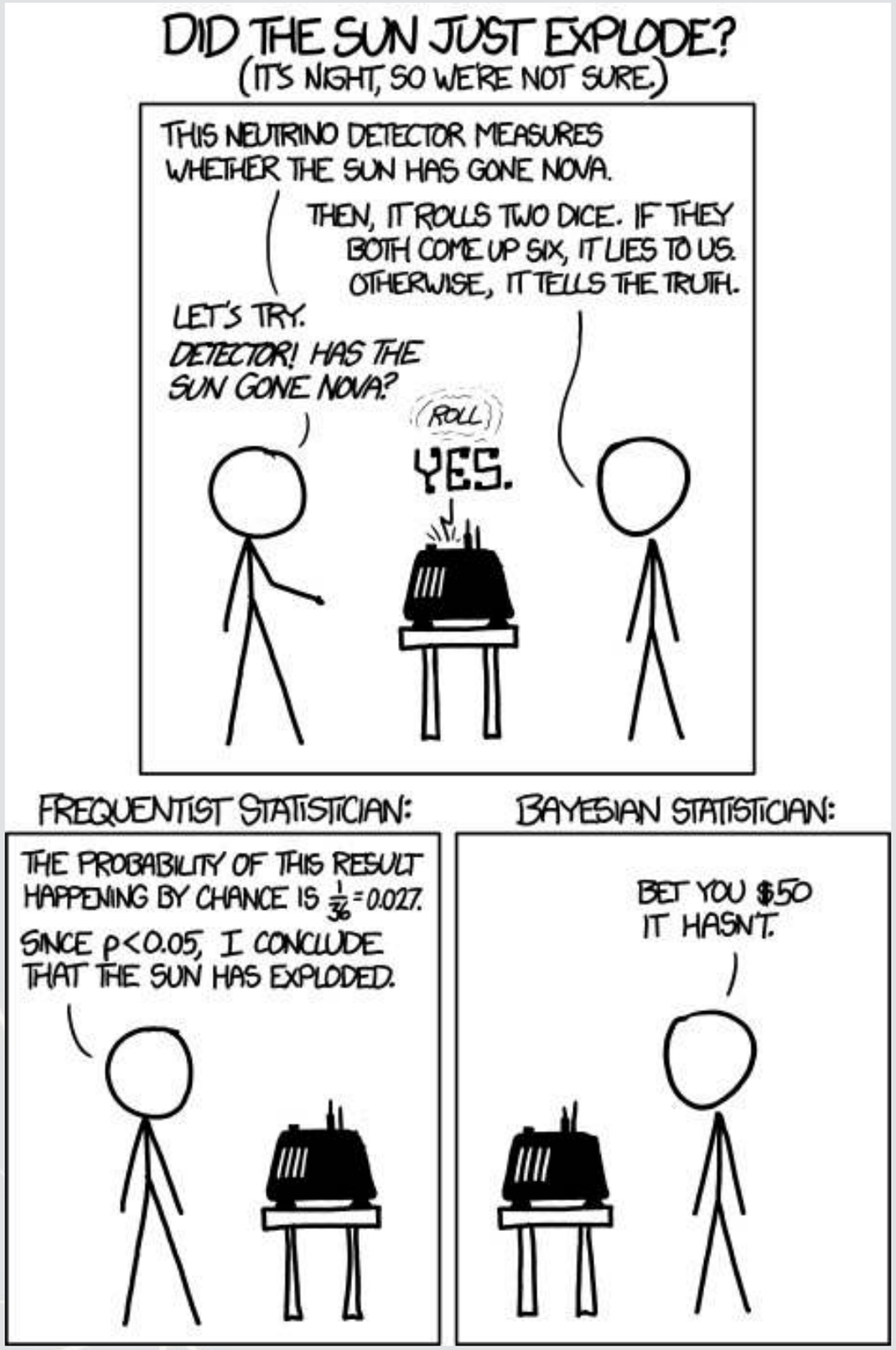
- The “correct” answer should occur most frequently, i.e., with a high probability
- Focus on true vs false
- Treat unknowns as fixed constants to figure out
  - Not random quantities
- Where it’s used
  - Classical statistics methods
    - Like OLS

# Bayesian statistics

> Focus on distributions and beliefs

- Prior distribution – what is believed before the experiment
- Posterior distribution: an updated belief of the distribution due to the experiment
- Derive distributions of parameters
- Where it’s used:
  - Many machine learning methods
    - Bayesian updating acts as the learning
  - Bayesian statistics

# Frequentist vs Bayesian methods



# Frequentist perspective: Repeat the test

```
detector <- function() {
  dice <- sample(1:6, size=2, replace=TRUE)
  if (sum(dice) == 12) {
    "exploded"
  } else {
    "still there"
  }
}

experiment <- replicate(1000,detector())
# p value
paste("p-value: ",
      sum(experiment == "still there") / 1000,
      "-- Reject H_A that sun exploded")
```

```
## [1] "p-value:  0.962 -- Reject H_A that sun exploded"
```

> Frequentist: The sun didn’t explode

# Bayes persepctive: Bayes rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- $A$: The sun exploded
- $B$: The detector said it exploded
- $P(A)$: Really, really small. Say, ~0.
- $P(B)$: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
- $P(B|A)$: $\frac{35}{36}$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{\frac{35}{36} \times \sim 0}{\frac{1}{36}} = 35 \times \sim 0 \approx 0$$

> Bayesian: The sun didn't explode

# What analytics typically relies on

- Regression approaches
  - Most often done in a frequentist manner
  - Can be done in a Bayesian manner as well
- Artificial Intelligence
  - Often frequentist
  - Sometimes neither – “It just works”
- Machine learning
  - Sometimes Bayesian, sometime frequentist
  - We’ll see both

> We will use both to some extent – for our purposes, we will not debate the merits of either school of thought, but use tools derived from both

# Confusion from frequentist approaches

- Possible contradictions:
  - $F$ test says the model is good yet nothing is statistically significant
  - Individual $p$-values are good yet the model isn't
  - One measure says the model is good yet another doesn't

> There are many ways to measure a model, each with their own merits. They don't always agree, and it's on us to pick a reasonable measure.

# Frequentist approaches to things

# Hypotheses

- $H_0$: The status quo is correct
  - Your proposed model doesn't work
- $H_A$: The model you are proposing works
- Frequentist statistics can never directly support $H_0$!
  - Only can fail to find support for $H_A$
- Even if our $p$-value is 1, we can't say that the results prove the null hypothesis!

# OLS terminology

- $y$: The output in our model
- $\hat{y}$: The *estimated* output in our model
- $x_i$: An input in our model
- $\hat{x}_i$: An *estimated* input in our model
- $\hat{}$: Something *estimated*
- $\alpha$: A constant, the expected value of $y$ when all $x_i$ are 0
- $\beta_i$: A coefficient on an input to our model
- $\varepsilon$: The error term
  - This is also the *residual* from the regression
    - What's left if you take actual $y$ minus the model prediction

# Regression

- Regression (like OLS) has the following assumptions
  1. The data is generated following some model
     - E.g., a linear model
       - Next week, a logistic model
  2. The data conforms to some statistical properties as required by the test
  3. The model coefficients are something to precisely determine
     - I.e., the coefficients are constants
  4. $p$-values provide a measure of the chance of an error in a particular aspect of the model
     - For instance, the p-value on $\beta_1$ in $y = \alpha + \beta_1 x_1 + \varepsilon$ essentially gives the probability that the sign of $\beta_1$ is wrong

# OLS Statistical properties

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \varepsilon$$

$$\hat{y} = \alpha + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \ldots + \hat{\varepsilon}$$

1. There should be a *linear* relationship between $y$ and each $x_i$
   - I.e., $y$ is [approximated by] a constant multiple of each $x_i$
   - Otherwise we **shouldn't** use a *linear* regression
2. Each $\hat{x}_i$ is normally distributed
   - Not so important with larger data sets, but a good to adhere to
3. Each observation is independent
   - We'll violate this one for the sake of *causality*
4. Homoskedasticity: Variance in errors is constant
   - This is important
5. Not too much multicollinearity
   - Each $\hat{x}_i$ should be relatively independent from the others
   - Some is OK

# Practical implications

> Models designed under a frequentist approach can only answer the question of “does this matter?”

- Is this a problem?
- Often, this is enough

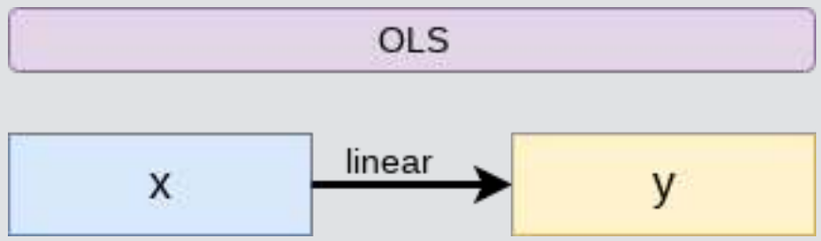
# Linear model implementation

# What exactly is a linear model?

- Anything OLS is linear
- Many transformations can be recast to linear
  - Ex.: $log(y) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 {x_1}^2 + \beta_4 x_1 \cdot x_2$
    - This is the same as $y' = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ where:
      - $y' = log(y)$
      - $x_3 = {x_1}^2$
      - $x_4 = x_1 \cdot x_2$

> Linear models are *very* flexible

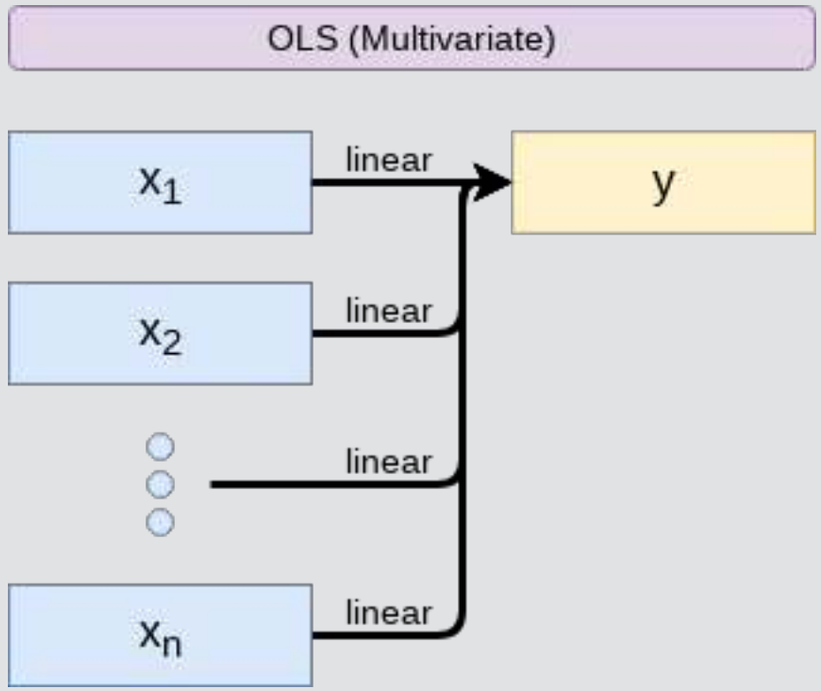
# Mental model of OLS: 1 input




> Simple OLS measures a simple linear relationship between an input and an output

- E.g.: Our first regression last week: Revenue on assets

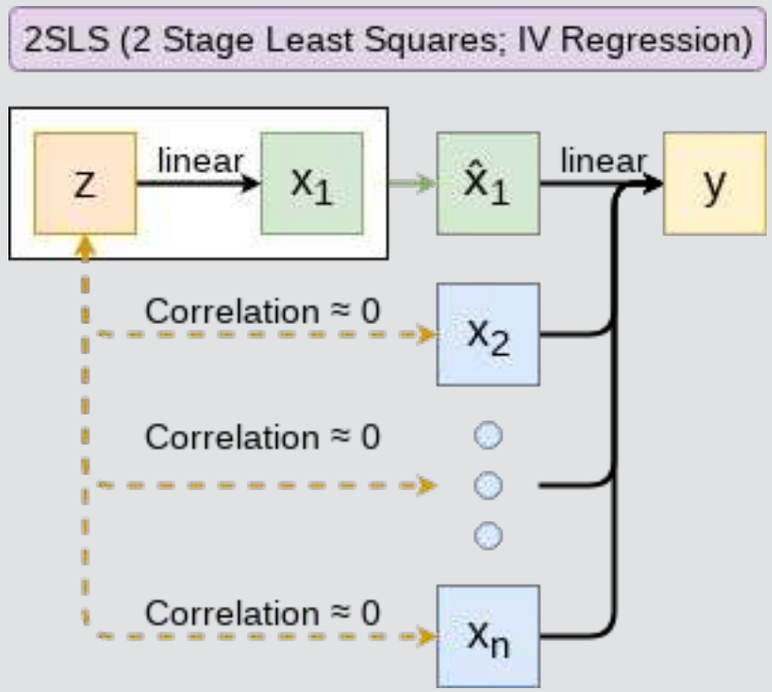
# Mental model of OLS: Multiple inputs




> OLS measures simple linear relationships between a set of inputs and one output

- E.g.: Our main models last week: Future revenue regressed on multiple accounting and macro variables

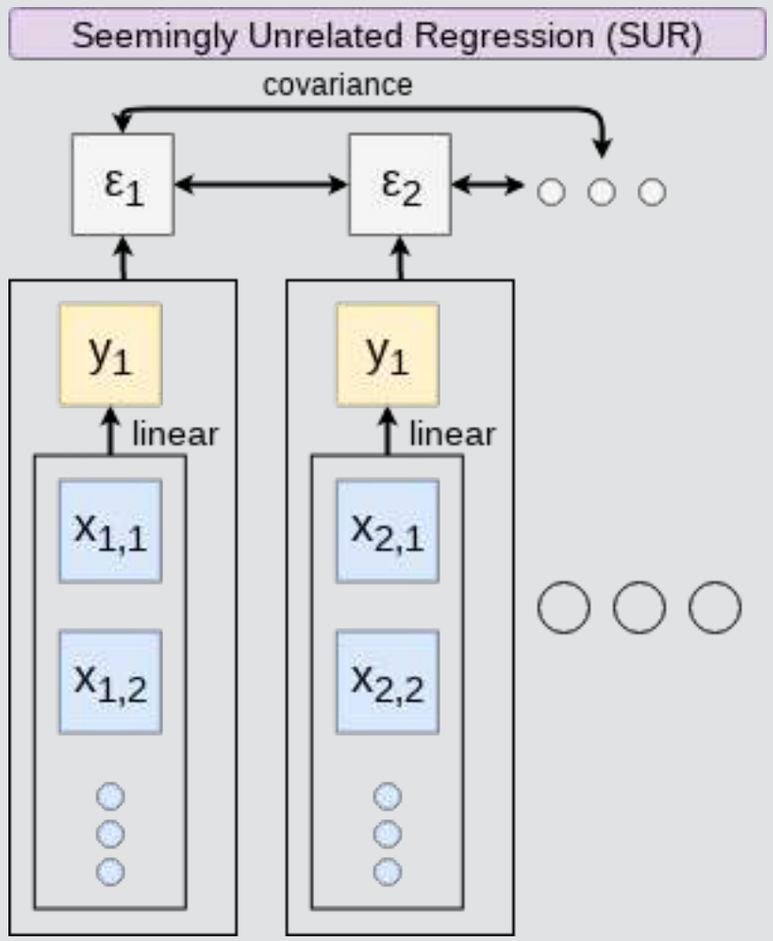
# Other linear models: IV Regression (2SLS)




> IV/2SLS models linear relationships where the effect of some $x_i$ on $y$ may be confounded by outside factors.

- E.g.: Modeling the effect of management pay duration (like bond duration) on firms' choice to issue earnings forecasts
  - Instrument with CEO tenure (Cheng, Cho, and Kim 2015)

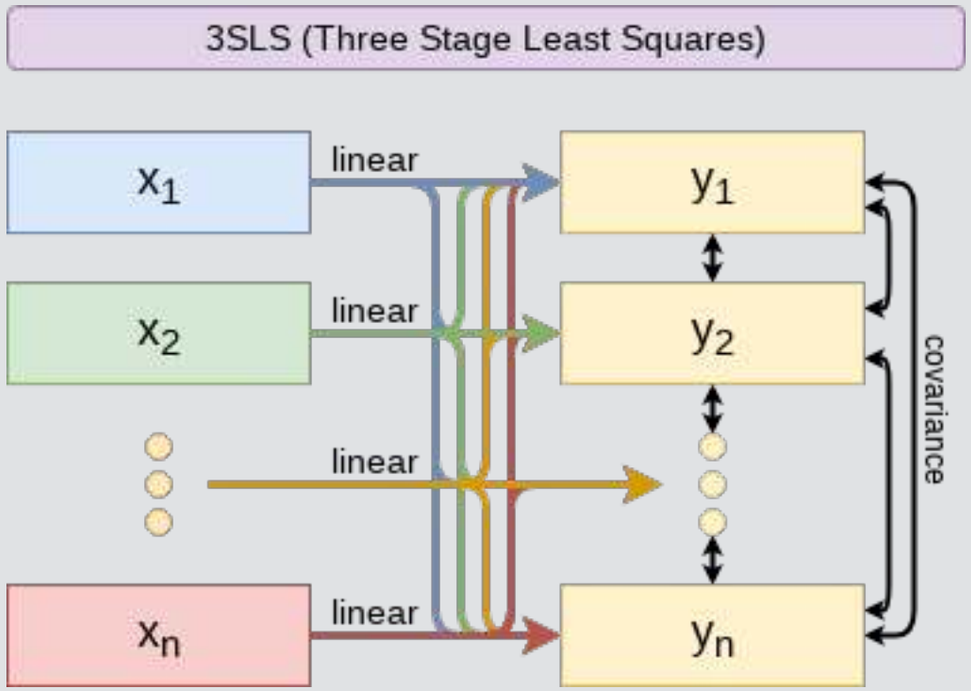
# Other linear models: SUR




SUR models systems with related error terms

- E.g.: Modeling both revenue and earnings simultaneously

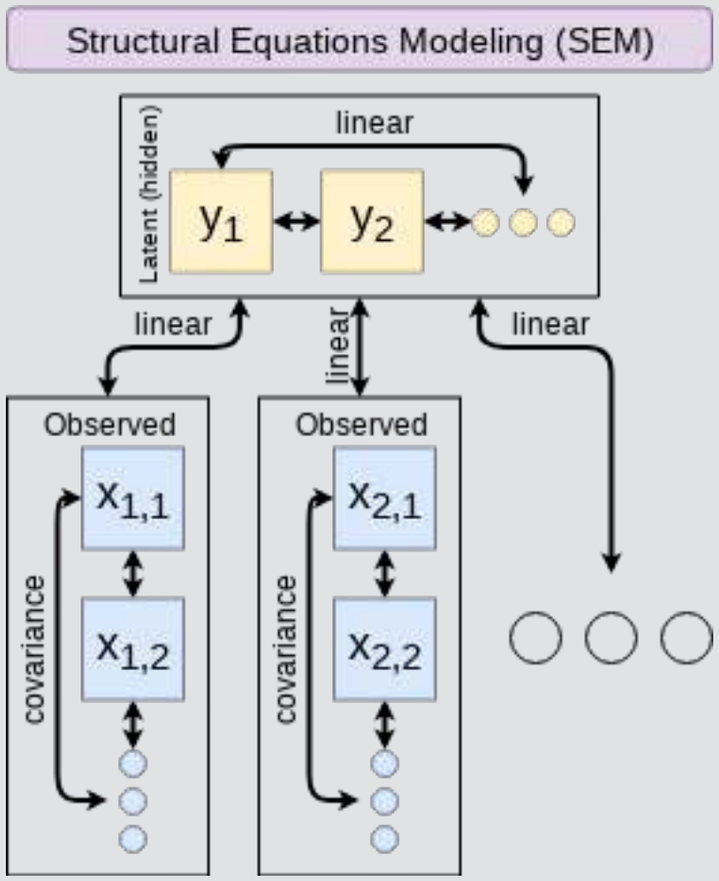
# Other linear models: 3SLS




> 3SLS models systems of equations with *related outputs*

- E.g.: Modeling both stock return, volatility, and volume simultaneously

# Other linear models: SEM




> SEM can model abstract and multi-level relationships

- E.g.: Showing that organizational commitment leads to higher job satisfaction, not the other way around (Poznanski and Bline 1999)

# Modeling choices: Model selection

> Pick what fits your problem!

- For forecasting a quantity
  - Usually some sort of linear model regressed using OLS
  - The other model types mentioned are great for simultaneous forecasting of multiple outputs
- For forecasting a binary outcome
  - Usually logit or a related model (we'll start this next week)
- For forensics:
  - Usually logit or a related model

# Modeling choices: Variable selection

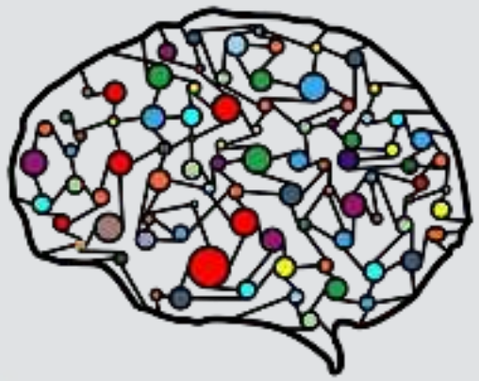
- The options:
  1. Use your own knowledge to select variables
  2. Use a selection model to automate it

Own knowledge

- Build a model based on your knowledge of the problem and situation
- This is generally better
  - The result should be more interpretable
  - For prediction, you should know relationships better than most algorithms

# Modeling choices: Automated selection

- Traditional methods include:
  - Forward selection: Start with nothing and add variables with the most contribution to Adj $R^2$ until it stops going up
  - Backward selection: Start with all inputs and remove variables with the worst (negative) contribution to Adj $R^2$ until it stops going up
  - Stepwise selection: Like forward selection, but drops non-significant predictors
- Newer methods:
  - Lasso and Elastic Net based models
    - Optimize with high penalties for complexity (i.e., # of inputs)
    - We will discuss these in week 6



# The overfitting problem

> Or: Why do we like simpler models so much?

- Overfitting happens when a model fits in-sample data *too well*…
  - To the point where it also models any idiosyncrasies or errors in the data
  - This harms prediction performance
    - Directly harming our forecasts

> An overfitted model works really well on its own data, and quite poorly on new data

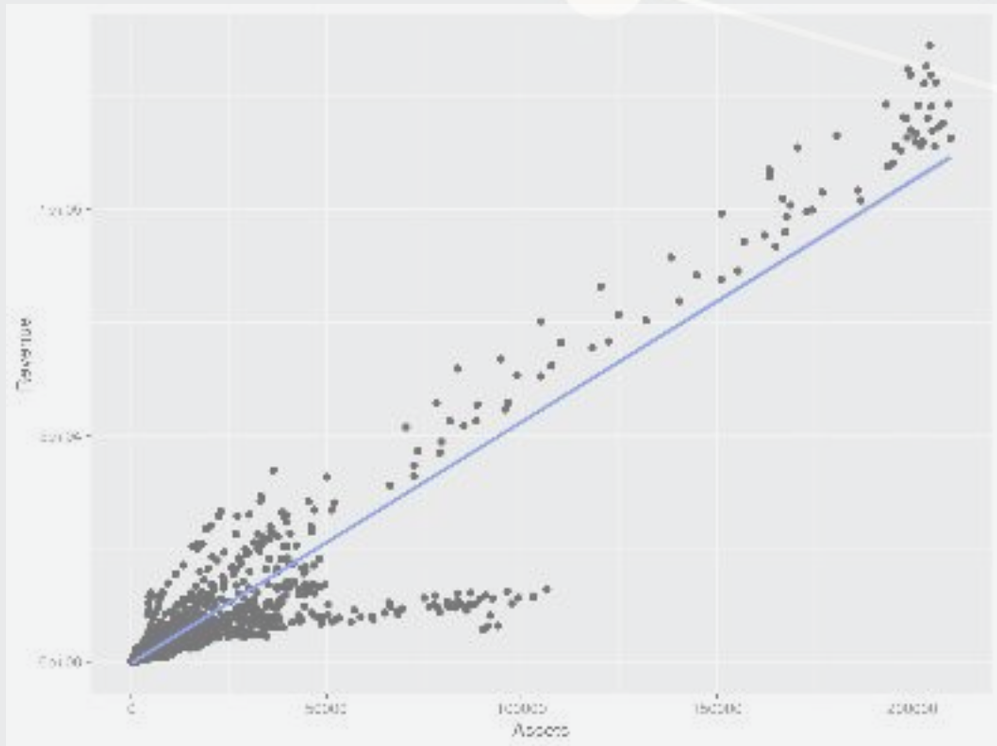
# Statistical tests and Interpretation

# Coefficients

- In OLS:

$$\beta_i$$

- A change in $x_i$ by 1 leads to a change in $y$ by $\beta_i$
- Essentially, the slope between $x$ and $y$
- The blue line in the chart is the regression line for $\hat{Revenue} = \alpha + \beta_i \hat{Assets}$ for retail firms since 1960



# P-values

- $p$-values tell us the probability that an individual result is due to random chance

> "The P value is defined as the probability under the assumption of no effect or no difference (null hypothesis), of obtaining a result equal to or more extreme than what was actually observed."" – Dahiru 2008

- These are very useful, particularly for a frequentist approach
- First used in the 1700s, but popularized by Ronald Fisher in the 1920s and 1930s

# P-values: Rule of thumb

- If $p < 0.05$ and the coefficient matches our mental model, we can consider this as supporting our model
  - If $p < 0.05$ but the coefficient is opposite, then it is suggesting a problem with our model
  - If $p > 0.10$, it is rejecting the alternative hypothesis
- If $0.05 < p < 0.10$ it depends…
  - For a small dataset or a complex problem, we can use $0.10$ as a cutoff
  - For a huge dataset or a simple problem, we should use $0.05$

# One vs two tailed tests

- Best practice:
  - **Use a two tailed test**
- Second best practice:
  - If you use a 1-tailed test, use a p-value cutoff of 0.025 or 0.05
    - This is equivalent to the best practice, just roundabout
- Common but semi-inappropriate:
  - Use a two tailed test with cutoffs of 0.05 or 0.10 because your hypothesis is directional

# $R^2$

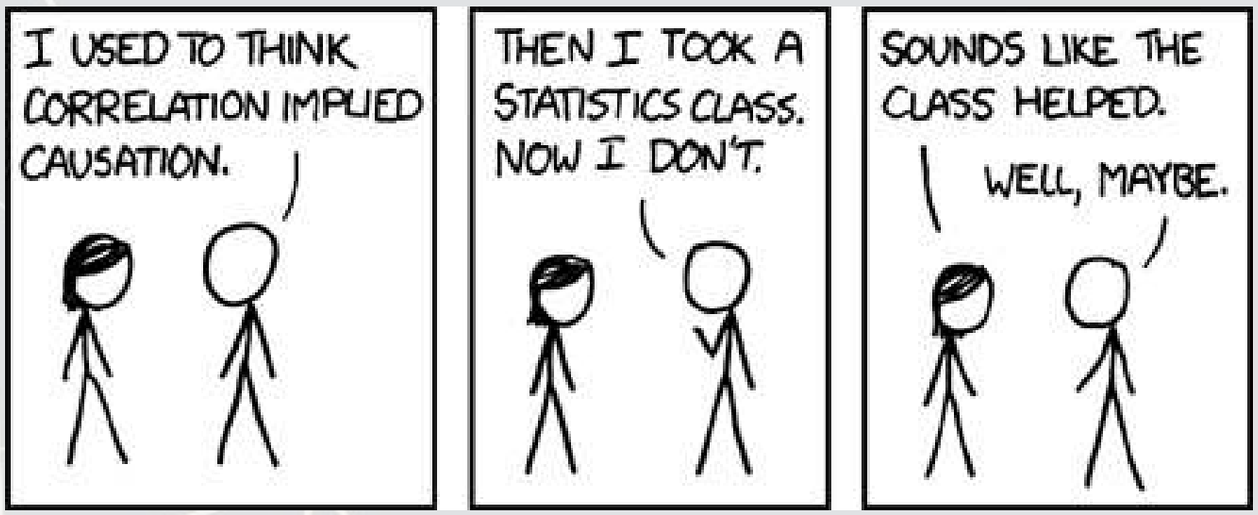
- $R^2$ = Explained variation / Total variation
  - Variation = difference in the observed output variable from its own mean
- A high $R^2$ indicates that the model fits the data very well
- A low $R^2$ indicates that the model is missing much of the variation in the output
- $R^2$ is technically a *biased* estimator
- Adjusted $R^2$ downweights $R^2$ and makes it unbiased
  - $R^2_{Adj} = PR^2 + 1 - P$
    - Where $P = \frac{n-1}{n-p-1}$
      - $n$ is the number of observations
      - $p$ is the number of inputs in the model

# Causality

# What is causality?

$$A \rightarrow B$$

- Causality is $A$ *causing* $B$
  - This means more than $A$ and $B$ are correlated
- I.e., If $A$ changes, $B$ changes. But $B$ changing doesn't mean $A$ changed
  - Unless $B$ is 100% driven by $A$
- Very difficult to determine, particularly for events that happen [almost] simultaneously
- Examples of correlations that aren't causation



# Time and causality

$$A \rightarrow B \text{ or } A \leftarrow B?$$

$$A_t \rightarrow B_{t+1}$$

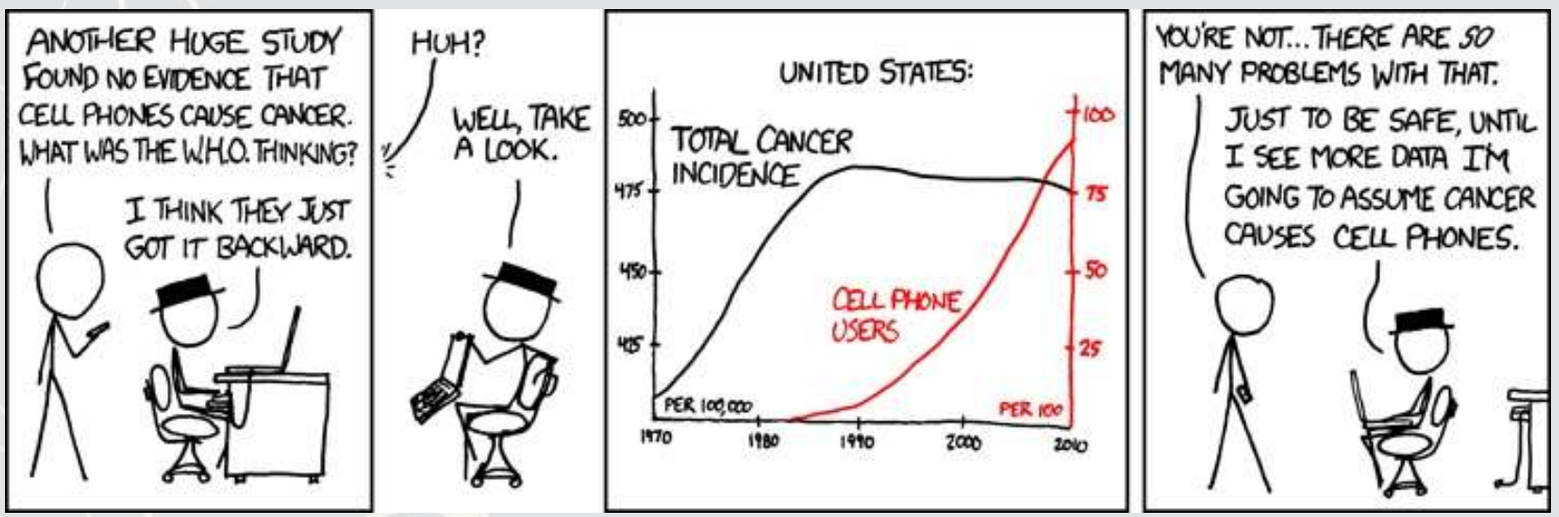
- If there is a separation in time, it's easier to say $A$ caused $B$
  - Observe $A$, then see if $B$ changes after
- Conveniently, we have this structure when forecasting
  - Recall last week's model:

$$Revenue_{t+1} = Revenue_t + \dots$$

# Time and causality break down

$A_t \rightarrow B_{t+1}$? OR $C \rightarrow A_t$ and $C \rightarrow B_{t+1}$?

- The above illustrates the *Correlated omitted variable problem*
  - $A$ doesn't cause $B$... Instead, some other force $C$ causes both
  - Bane of social scientists everywhere
- This is less important for predictive analytics, as we care more about performance, but...
- It can complicate interpreting your results
- Figuring out $C$ can help improve you model's predictions
  - So find C!



# Today's application: Quarterly retail revenue

# The question

> How can we predict quarterly revenue for retail companies, leveraging our knowledge of such companies

- In aggregate
- By Store
- By department

# More specifically…

- Consider time dimensions
  - What matters:
    - Last quarter?
    - Last year?
    - Other timeframes?
  - Cyclicality

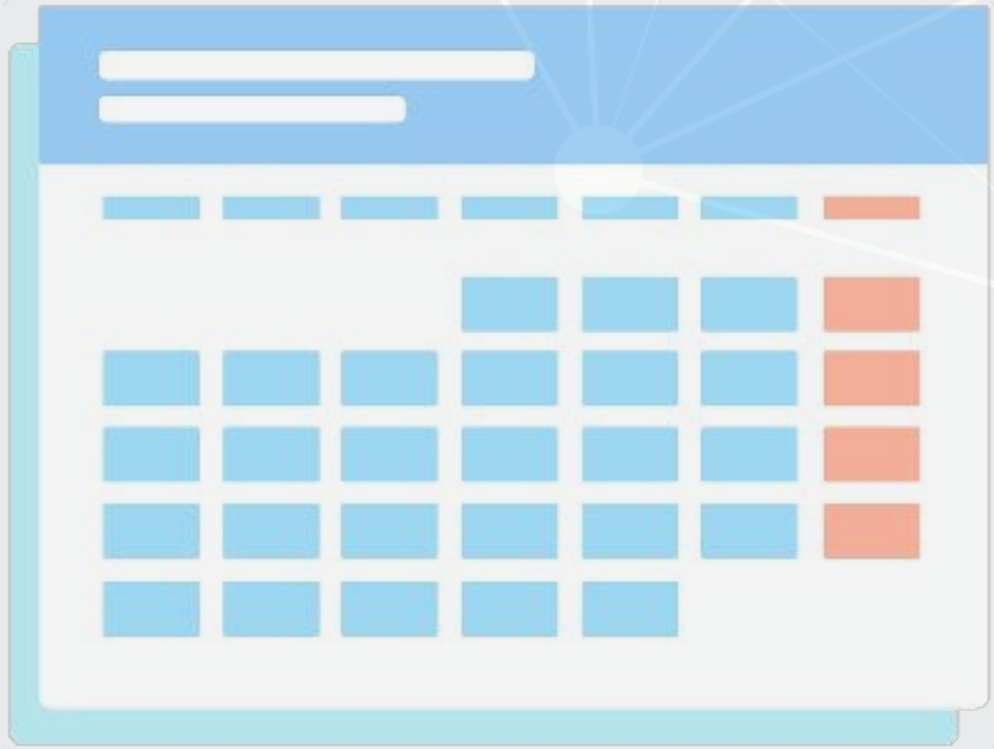
# Time and OLS

# Time matters a lot for retail


Great Singapore Sale

# How to capture time effects?

- Autoregression
  - Regress $y_t$ on earlier value(s) of itself
    - Last quarter, last year, etc.
- Controlling for time directly in the model
  - Essentially the same as fixed effects last week



# Quarterly revenue prediction

# The data

- From quarterly reports
- Two sets of firms:
  - US “Hypermarkets & Super Centers” (GICS: 30101040)
  - US “Multiline Retail” (GICS: 255030)
- Data from Compustat - Capital IQ > North America - Daily > Fundamentals Quarterly

# Formalization

1. Question
   - How can we predict quarterly revenue for large retail companies?
2. Hypothesis (just the alternative ones)
   1. Current quarter revenue helps predict next quarter revenue
   2. 3 quarters ago revenue helps predict next quarter revenue (Year-over-year)
   3. Different quarters exhibit different patterns (seasonality)
   4. A long-run autoregressive model helps predict next quarter revenue
3. Prediction
   - Use OLS for all the above – t-tests for coefficients
   - Hold out sample: 2015-2017

# Variable generation

```r
library(tidyverse)  # As always
library(plotly)  # interactive graphs
library(lubridate)  # import some sensible date functions

# Generate quarter over quarter growth "revtq_gr"
df <- df %>% group_by(gvkey) %>% mutate(revtq_gr=revtq / lag(revtq) - 1) %>% ungroup()

# Generate year-over-year growth "revtq_yoy"
df <- df %>% group_by(gvkey) %>% mutate(revtq_yoy=revtq / lag(revtq, 4) - 1) %>% ungroup()

# Generate first difference "revtq_d"
df <- df %>% group_by(gvkey) %>% mutate(revtq_d=revtq - lag(revtq)) %>% ungroup()

# Generate a proper date
# Date was YYMMDDs10: YYYY/MM/DD, can be converted from text to date easily
df$date <- as.Date(df$datadate)  # Built in to R
```

- Use mutate for variables using lags
- as.Date() can take a date formatted as “YYYY/MM/DD” and convert to a proper date value
  - You can convert other date types using the format= argument
    - i.e., “DD.MM.YYYY” is format code “%d.%m.%Y”
    - Full list of date codes

# Example output

| conm | date | revtq | revtq_gr | revtq_yoy | revtq_d |
|---|---|---|---|---|---|
| ALLIED STORES | 1962-04-30 | 156.5 | NA | NA | NA |
| ALLIED STORES | 1962-07-31 | 161.9 | 0.0345048 | NA | 5.4 |
| ALLIED STORES | 1962-10-31 | 176.9 | 0.0926498 | NA | 15.0 |
| ALLIED STORES | 1963-01-31 | 275.5 | 0.5573770 | NA | 98.6 |
| ALLIED STORES | 1963-04-30 | 171.1 | -0.3789474 | 0.0932907 | -104.4 |
| ALLIED STORES | 1963-07-31 | 182.2 | 0.0648743 | 0.1253860 | 11.1 |

```
## # A tibble: 6 x 3
##   conm          date       datadate
##   <chr>         <date>     <chr>
## 1 ALLIED STORES 1962-04-30 1962/04/30
## 2 ALLIED STORES 1962-07-31 1962/07/31
## 3 ALLIED STORES 1962-10-31 1962/10/31
## 4 ALLIED STORES 1963-01-31 1963/01/31
## 5 ALLIED STORES 1963-04-30 1963/04/30
## 6 ALLIED STORES 1963-07-31 1963/07/31
```

# Create 8 quarters (2 years) of lags

```
# Custom Function to generate a series of lags
multi_lag <- function(df, lags, var, ext="") {
  lag_names <- paste0(var,ext,lags)
  lag_funs <- setNames(paste("dplyr::lag(.,",lags,")"), lag_names)
  df %>% group_by(gvkey) %>% mutate_at(vars(var), funs_(lag_funs)) %>% ungroup()
}
# Generate lags "revtq_l#"
df <- multi_lag(df, 1:8, "revtq", "_l")

# Generate changes "revtq_gr#"
df <- multi_lag(df, 1:8, "revtq_gr")

# Generate year-over-year changes "revtq_yoy#"
df <- multi_lag(df, 1:8, "revtq_yoy")

# Generate first differences "revtq_d#"
df <- multi_lag(df, 1:8, "revtq_d")

# Equivalent brute force code for this is in the appendix
```

- paste0(): creates a string vector by concatenating all inputs
- paste(): same as paste0(), but with spaces added in between
- setNames(): allows for storing a value and name simultaneously
- mutate_at(): is like mutate but with a list of functions

# Example output

| conm | date | revtq | revtq_l1 | revtq_l2 | revtq_l3 | revtq_l4 |
|---|---|---|---|---|---|---|
| ALLIED STORES | 1962-04-30 | 156.5 | NA | NA | NA | NA |
| ALLIED STORES | 1962-07-31 | 161.9 | 156.5 | NA | NA | NA |
| ALLIED STORES | 1962-10-31 | 176.9 | 161.9 | 156.5 | NA | NA |
| ALLIED STORES | 1963-01-31 | 275.5 | 176.9 | 161.9 | 156.5 | NA |
| ALLIED STORES | 1963-04-30 | 171.1 | 275.5 | 176.9 | 161.9 | 156.5 |
| ALLIED STORES | 1963-07-31 | 182.2 | 171.1 | 275.5 | 176.9 | 161.9 |

# Clean and split into training and testing

```
# Clean the data: Replace NaN, Inf, and -Inf with NA
df <- df %>%
  mutate_if(is.numeric, funs(replace(., !is.finite(.), NA)))

# Split into training and testing data
# Training data: We'll use data released before 2015
train <- filter(df, year(date) < 2015)

# Testing data: We'll use data released 2015 through 2018
test <- filter(df, year(date) >= 2015)
```

- Same cleaning function as last week:
  - Replaces all NaN, Inf, and -Inf with NA
- year() comes from lubridate

# Univariate stats

# Univariate stats

- To get a better grasp on the problem, looking at univariate stats can help
  - Summary stats (using `summary()`)
  - Correlations using `cor()`
  - Plots using your preferred package such as `ggplot2`

```
summary(df[,c("revtq","revtq_gr","revtq_yoy", "revtq_d","fqtr")])
```

```
##      revtq             revtq_gr          revtq_yoy
##  Min.   :     0.00   Min.   :-1.0000   Min.   :-1.0000
##  1st Qu.:    64.46   1st Qu.:-0.1112   1st Qu.: 0.0077
##  Median :   273.95   Median : 0.0505   Median : 0.0740
##  Mean   :  2439.38   Mean   : 0.0650   Mean   : 0.1273
##  3rd Qu.:  1254.21   3rd Qu.: 0.2054   3rd Qu.: 0.1534
##  Max.   :136267.00   Max.   :14.3333   Max.   :47.6600
##  NA's   :367         NA's   :690       NA's   :940
##     revtq_d              fqtr
##  Min.   :-24325.21   Min.   :1.000
##  1st Qu.:   -19.33   1st Qu.:1.000
##  Median :     4.30   Median :2.000
##  Mean   :    22.66   Mean   :2.478
##  3rd Qu.:    55.02   3rd Qu.:3.000
##  Max.   : 15495.00   Max.   :4.000
##  NA's   :663
```

# ggplot2 for visualization

- The next slides will use some custom functions using ggplot2
- ggplot2 has an odd syntax:
  - It doesn't use pipes (%>%), but instead adds everything together (+)

```
library(ggplot2)  # or tidyverse -- it's part of tidyverse
df %>%
  ggplot(aes(y=var_for_y_axis, x=var_for_y_axis)) +
  geom_point()  # scatterplot
```

- aes() is for aesthetics – how the chart is set up
- Other useful aesthetics:
  - group= to set groups to list in the legend. Not needed if using the below though
  - color= to set color by some grouping variable. Put factor() around the variable if you want discrete groups, otherwise it will do a color scale (light to dark)
  - shape= to set shapes for points – see here for a list

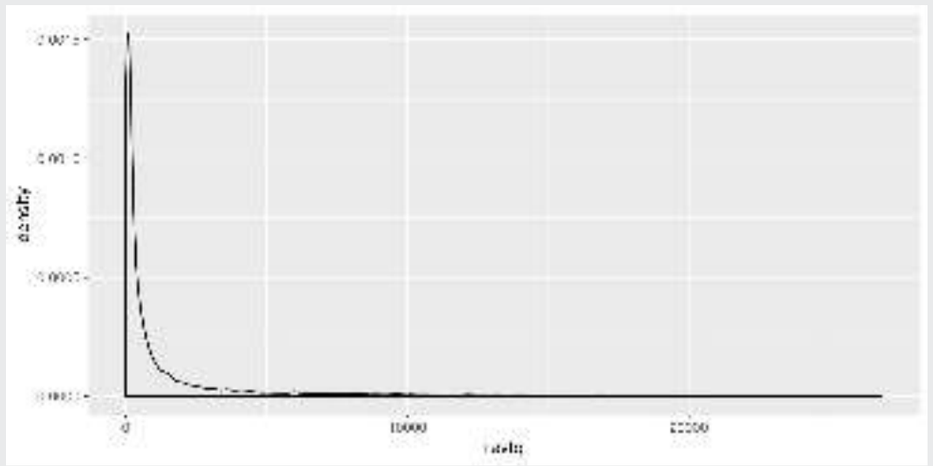
# ggplot2 for visualization

```
library(ggplot2)  # or tidyverse -- it's part of tidyverse
df %>%
  ggplot(aes(y=var_for_y_axis, x=var_for_y_axis)) +
  geom_point()  # scatterplot
```

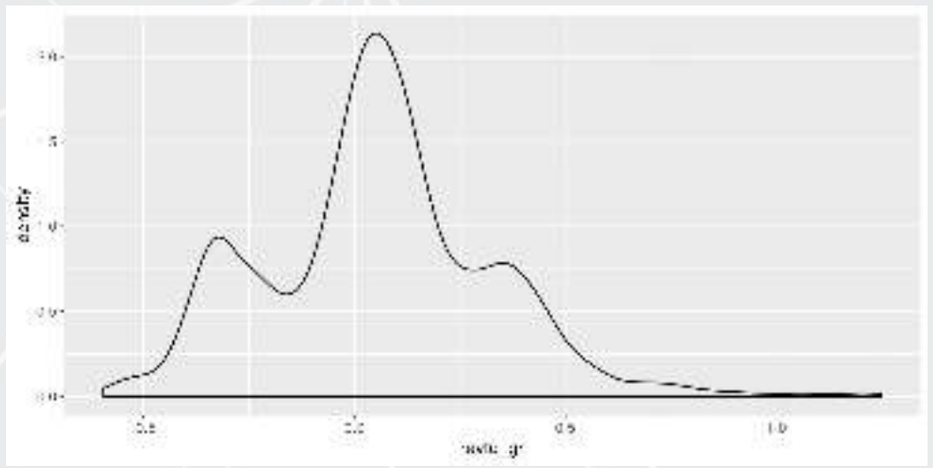
- geom stands for geometry – any shapes, lines, etc. start with geom
- Other useful geoms:
  - geom_line(): makes a line chart
  - geom_bar(): makes a bar chart – y is the height, x is the category
  - geom_smooth(method="lm"): Adds a linear regression into the chart
  - geom_abline(slope=1): Adds a 45° line
- Add xlab("Label text here") to change the x-axis label
- Add ylab("Label text here") to change the y-axis label
- Add ggtitle("Title text here") to add a title
- Plenty more details in the 'Data Visualization Cheat Sheet' on eLearn

# Plotting: Distribution of revenue

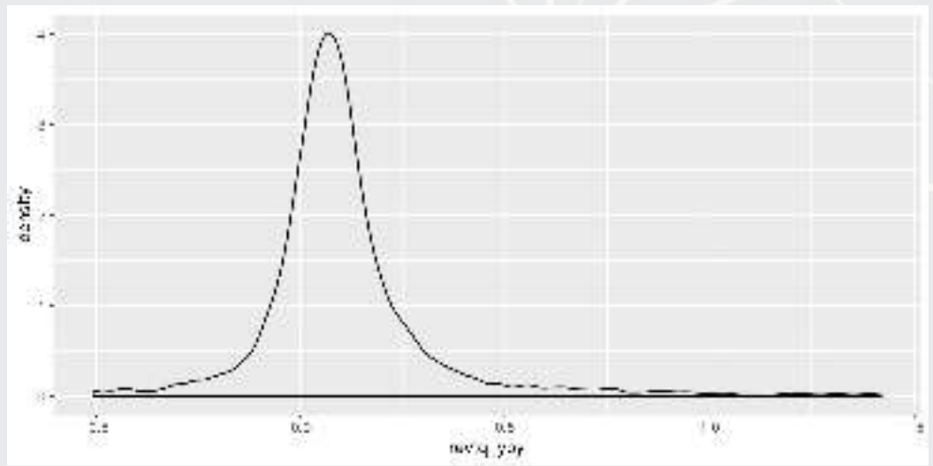
1. Revenue



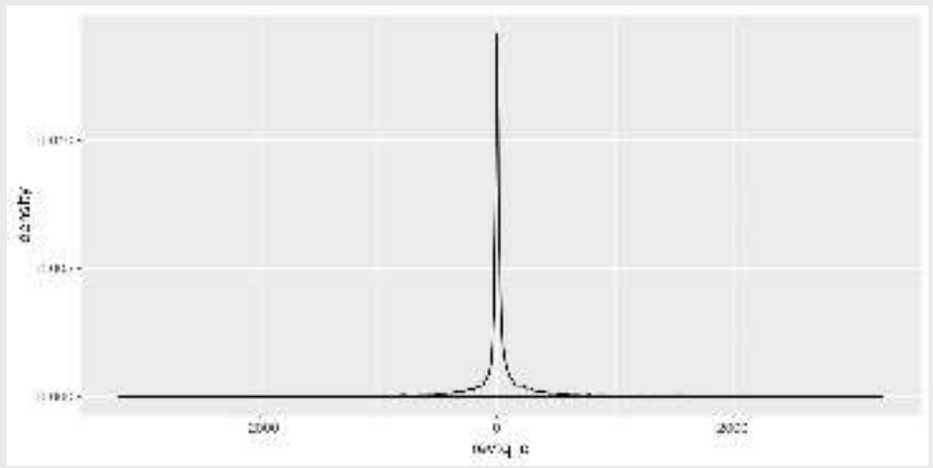
2. Quarterly growth



3. Year-over-year growth



4. First difference



# What do we learn from these graphs?

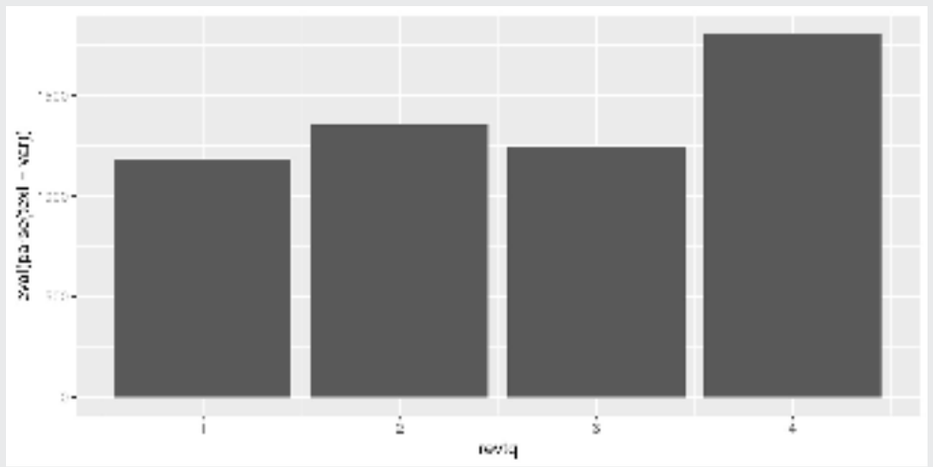
1. Revenue
   - 
2. Quarterly growth
   - 
3. Year-over-year growth
   - 
4. First difference
   - 

# What do we learn from these graphs?

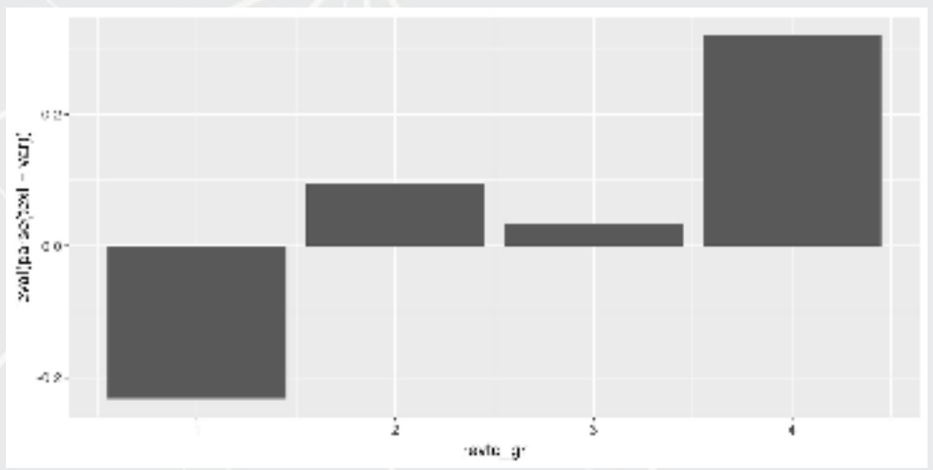
1. Revenue
   - This is really skewed data – a lot of small revenue quarters, but a significant amount of large revenue quarters in the tail
     - Potential fix: use `log(revtq)`?
2. Quarterly growth
   - Quarterly growth is reasonably close to normally distributed
     - Good for OLS
3. Year-over-year growth
   - Year over year growth is reasonably close to normally distributed
     - Good for OLS
4. First difference
   - Reasonably close to normally distributed, with really long tails
     - Good enough for OLS

# Plotting: Mean revenue by quarter

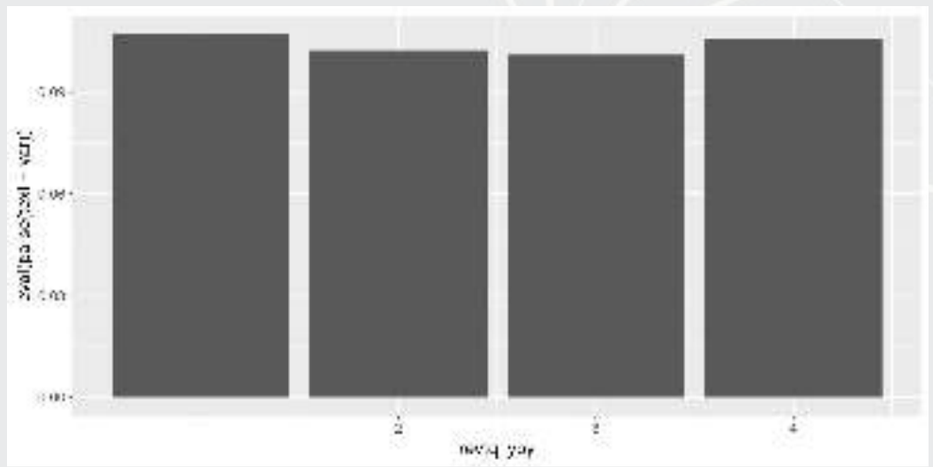
1. Revenue



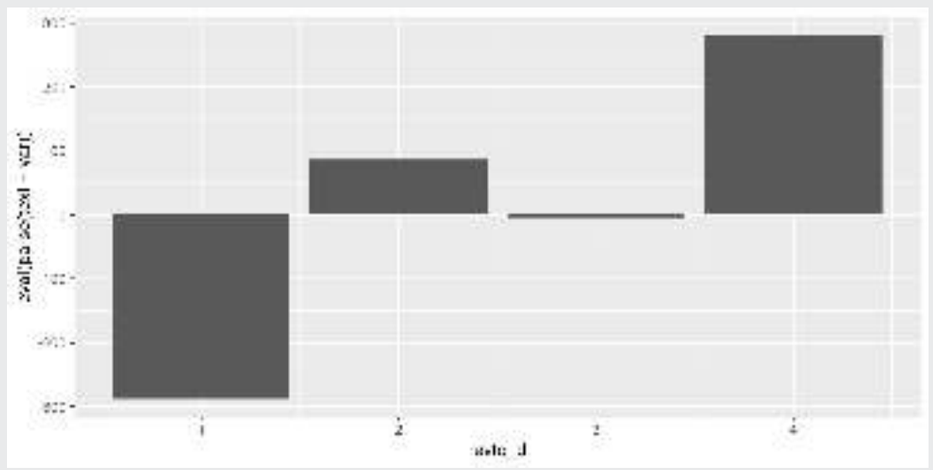
2. Quarterly growth



3. Year-over-year growth



4. First difference



# What do we learn from these graphs?

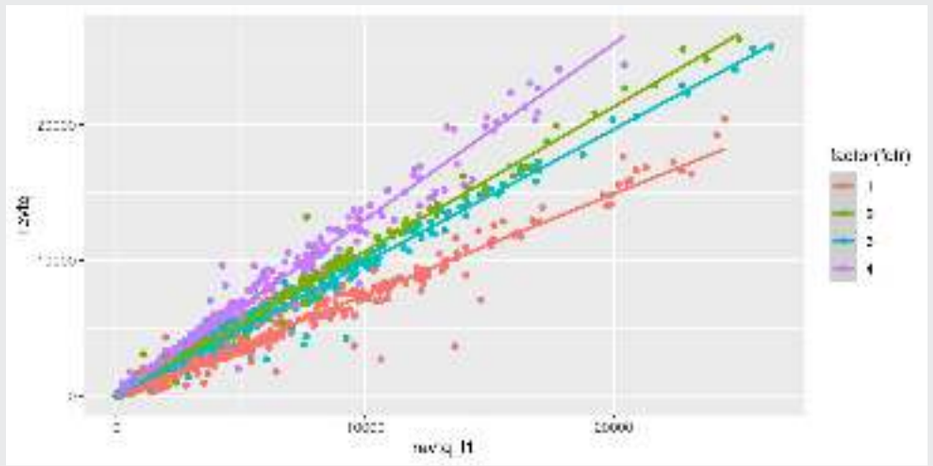
1. Revenue
   - 
2. Quarterly growth
   - 
3. Year-over-year growth
   - 
4. First difference
   - 

# What do we learn from these graphs?

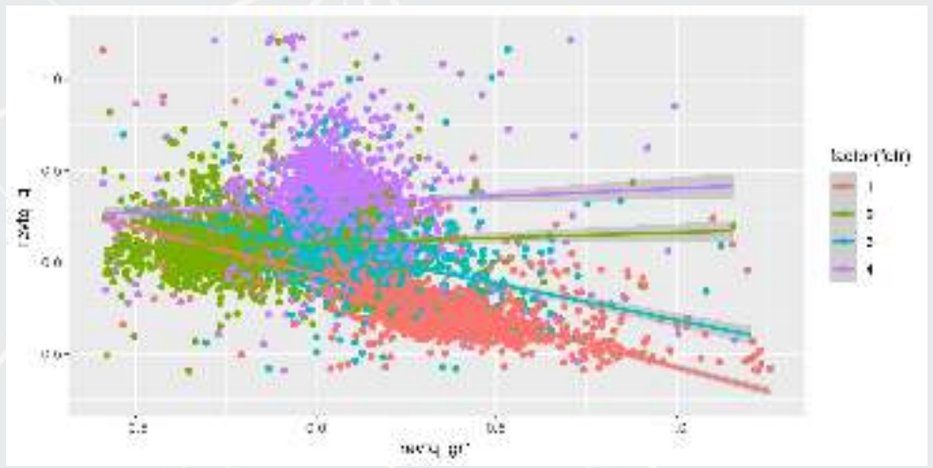
1. Revenue
   - Revenue seems cyclical!
2. Quarterly growth
   - Definitely cyclical!
3. Year-over-year growth
   - Year over year difference is less cyclical – more constant
4. First difference
   - Definitely cyclical!

# Plotting: Revenue vs lag by quarter

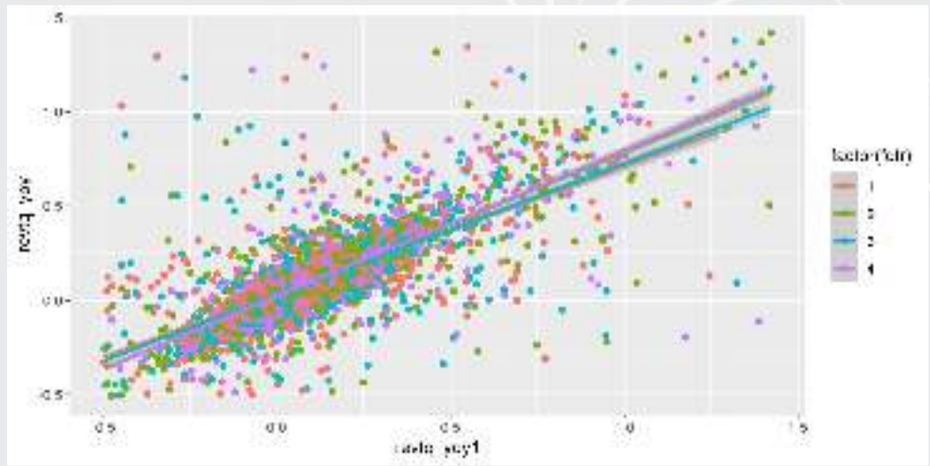
1. Revenue



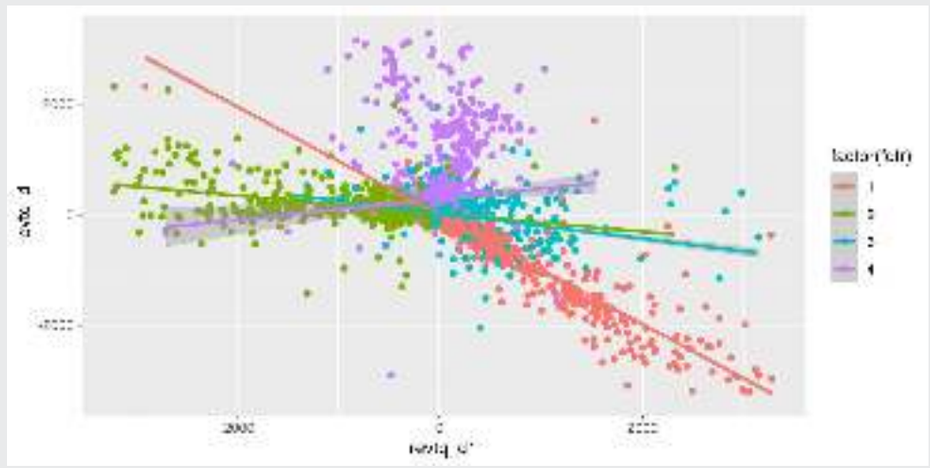

2. Quarterly growth




3. Year-over-year growth




4. First difference



# What do we learn from these graphs?

1. Revenue
   - Revenue is really linear! But each quarter has a distinct linear relation.
2. Quarterly growth
   - All over the place. Each quarter appears to have a different pattern though. Quarters will matter.
3. Year-over-year growth
   - Linear but noisy.
4. First difference
   - Again, all over the place. Each quarter appears to have a different pattern though. Quarters will matter.

# Correlation matrices

```
cor(train[,c("revtq","revtq_l1","revtq_l2","revtq_l3", "revtq_l4")],
    use="complete.obs")
```

```
##              revtq  revtq_l1  revtq_l2  revtq_l3  revtq_l4
## revtq    1.0000000 0.9916167 0.9938489 0.9905522 0.9972735
## revtq_l1 0.9916167 1.0000000 0.9914767 0.9936977 0.9898184
## revtq_l2 0.9938489 0.9914767 1.0000000 0.9913489 0.9930152
## revtq_l3 0.9905522 0.9936977 0.9913489 1.0000000 0.9906006
## revtq_l4 0.9972735 0.9898184 0.9930152 0.9906006 1.0000000
```

```
cor(train[,c("revtq_gr","revtq_gr1","revtq_gr2","revtq_gr3", "revtq_gr4")],
    use="complete.obs")
```

```
##              revtq_gr   revtq_gr1   revtq_gr2   revtq_gr3   revtq_gr4
## revtq_gr    1.00000000 -0.32291329  0.06299605 -0.22769442  0.64570015
## revtq_gr1 -0.32291329  1.00000000 -0.31885020  0.06146805 -0.21923630
## revtq_gr2  0.06299605 -0.31885020  1.00000000 -0.32795121  0.06775742
## revtq_gr3 -0.22769442  0.06146805 -0.32795121  1.00000000 -0.31831023
## revtq_gr4  0.64570015 -0.21923630  0.06775742 -0.31831023  1.00000000
```

> Retail revenue has really high autocorrelation! Concern for multicolinearity. Revenue growth is less autocorrelated and oscillates.

# Correlation matrices

```
cor(train[,c("revtq_yoy","revtq_yoy1","revtq_yoy2","revtq_yoy3", "revtq_yoy4")],
    use="complete.obs")
```

```
##             revtq_yoy revtq_yoy1 revtq_yoy2 revtq_yoy3 revtq_yoy4
## revtq_yoy   1.0000000  0.6554179  0.4127263  0.4196003  0.1760055
## revtq_yoy1 0.6554179  1.0000000  0.5751128  0.3665961  0.3515105
## revtq_yoy2 0.4127263  0.5751128  1.0000000  0.5875643  0.3683539
## revtq_yoy3 0.4196003  0.3665961  0.5875643  1.0000000  0.5668211
## revtq_yoy4 0.1760055  0.3515105  0.3683539  0.5668211  1.0000000
```

```
cor(train[,c("revtq_d","revtq_d1","revtq_d2","revtq_d3", "revtq_d4")],
    use="complete.obs")
```

```
##             revtq_d   revtq_d1   revtq_d2   revtq_d3   revtq_d4
## revtq_d    1.0000000 -0.6181516  0.3309349 -0.6046998  0.9119911
## revtq_d1 -0.6181516  1.0000000 -0.6155259  0.3343317 -0.5849841
## revtq_d2  0.3309349 -0.6155259  1.0000000 -0.6191366  0.3165450
## revtq_d3 -0.6046998  0.3343317 -0.6191366  1.0000000 -0.5864285
## revtq_d4  0.9119911 -0.5849841  0.3165450 -0.5864285  1.0000000
```

> Year over year change fixes the multicollinearity issue. First difference oscillates like quarter over quarter growth.

# R Practice

- This practice will look at predicting Walmart's quarterly revenue using:
  - 1 lag
  - Cyclicality
- Practice using:
  - `mutate()`
  - `lm()`
  - `ggplot2`
- Do the exercises in today's practice file
  - R Practice
  - Shortlink: rmc.link/420r4

# Forecasting

# 1 period models

1. 1 Quarter lag
   - We saw a very strong linear pattern here earlier

```
mod1 <- lm(revtq ~ revtq_l1, data=train)
```

2. Quarter and year lag
   - Year-over-year seemed pretty constant

```
mod2 <- lm(revtq ~ revtq_l1 + revtq_l4, data=train)
```

3. 2 years of lags
   - Other lags could also help us predict

```
mod3 <- lm(revtq ~ revtq_l1 + revtq_l2 + revtq_l3 + revtq_l4 +
           revtq_l5 + revtq_l6 + revtq_l7 + revtq_l8, data=train)
```

4. 2 years of lags, by observation quarter
   - Take into account cyclicality observed in bar charts

```
mod4 <- lm(revtq ~ (revtq_l1 + revtq_l2 + revtq_l3 + revtq_l4 +
           revtq_l5 + revtq_l6 + revtq_l7 + revtq_l8):factor(fqtr),
         data=train)
```

# Quarter lag

```
summary(mod1)
```

```
##
## Call:
## lm(formula = revtq ~ revtq_l1, data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -24438.7    -34.0    -11.7     34.6  15200.5
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 15.639975  13.514877   1.157    0.247
## revtq_l1     1.003038   0.001556 644.462   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1152 on 7676 degrees of freedom
##   (662 observations deleted due to missingness)
## Multiple R-squared:  0.9819, Adjusted R-squared:  0.9819
## F-statistic: 4.153e+05 on 1 and 7676 DF,  p-value: < 2.2e-16
```

# Quarter and year lag

```
summary(mod2)
```

```
##
## Call:
## lm(formula = revtq ~ revtq_l1 + revtq_l4, data = train)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -20245.7    -18.4     -4.4     19.1   9120.8
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.444986   7.145633   0.762    0.446
## revtq_l1    0.231759   0.005610  41.312   <2e-16 ***
## revtq_l4    0.815570   0.005858 139.227   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 592.1 on 7274 degrees of freedom
##   (1063 observations deleted due to missingness)
## Multiple R-squared:  0.9954, Adjusted R-squared:  0.9954
## F-statistic: 7.94e+05 on 2 and 7274 DF,  p-value: < 2.2e-16
```

# 2 years of lags

```
summary(mod3)
```

```
##
## Call:
## lm(formula = revtq ~ revtq_l1 + revtq_l2 + revtq_l3 + revtq_l4 +
##     revtq_l5 + revtq_l6 + revtq_l7 + revtq_l8, data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5005.6   -12.9    -3.7     9.3  5876.3
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.02478    4.37003   0.921    0.3571
## revtq_l1     0.77379    0.01229  62.972  < 2e-16 ***
## revtq_l2     0.10497    0.01565   6.707 2.16e-11 ***
## revtq_l3    -0.03091    0.01538  -2.010    0.0445 *
## revtq_l4     0.91982    0.01213  75.800  < 2e-16 ***
## revtq_l5    -0.76459    0.01324 -57.749  < 2e-16 ***
## revtq_l6    -0.08080    0.01634  -4.945 7.80e-07 ***
## revtq_l7     0.01146    0.01594   0.719    0.4721
## revtq_l8     0.07924    0.01209   6.554 6.03e-11 ***
```

# 2 years of lags, by observation quarter

```
summary(mod4)
```

```
##
## Call:
## lm(formula = revtq ~ (revtq_l1 + revtq_l2 + revtq_l3 + revtq_l4 +
##     revtq_l5 + revtq_l6 + revtq_l7 + revtq_l8):factor(fqtr),
##     data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6066.6   -13.9     0.1    15.1  4941.1
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)            -0.201107   4.004046  -0.050 0.959944
## revtq_l1:factor(fqtr)1  0.488584   0.021734  22.480  < 2e-16 ***
## revtq_l1:factor(fqtr)2  1.130563   0.023017  49.120  < 2e-16 ***
## revtq_l1:factor(fqtr)3  0.774983   0.028727  26.977  < 2e-16 ***
## revtq_l1:factor(fqtr)4  0.977353   0.026888  36.349  < 2e-16 ***
## revtq_l2:factor(fqtr)1  0.258024   0.035136   7.344 2.33e-13 ***
## revtq_l2:factor(fqtr)2 -0.100284   0.024664  -4.066 4.84e-05 ***
## revtq_l2:factor(fqtr)3  0.212954   0.039698   5.364 8.40e-08 ***
```

# Testing out of sample

- RMSE: Root mean square Error
- RMSE is very affected by outliers, and a bad choice for noisy data that you are OK with missing a few outliers here and there
  - Doubling error *quadruples* that part of the error

```
rmse <- function(v1, v2) {
  sqrt(mean((v1 - v2)^2, na.rm=T))
}
```

  - MAE: Mean absolute error
  - MAE is measures average accuracy with no weighting
    - Doubling error *doubles* that part of the error

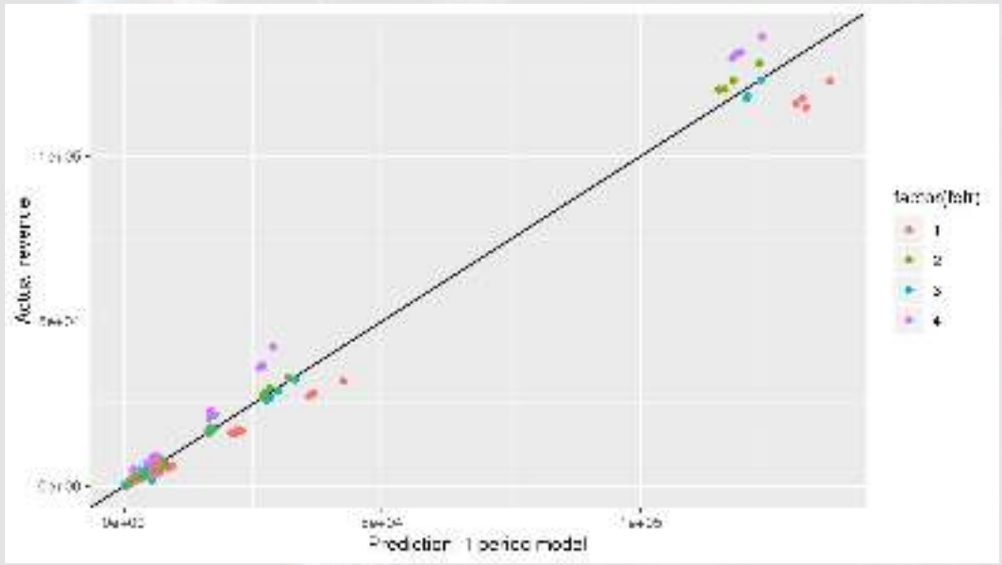
```
mae <- function(v1, v2) {
  mean(abs(v1-v2), na.rm=T)
}
```

> Both are commonly used for evaluating OLS out of sample

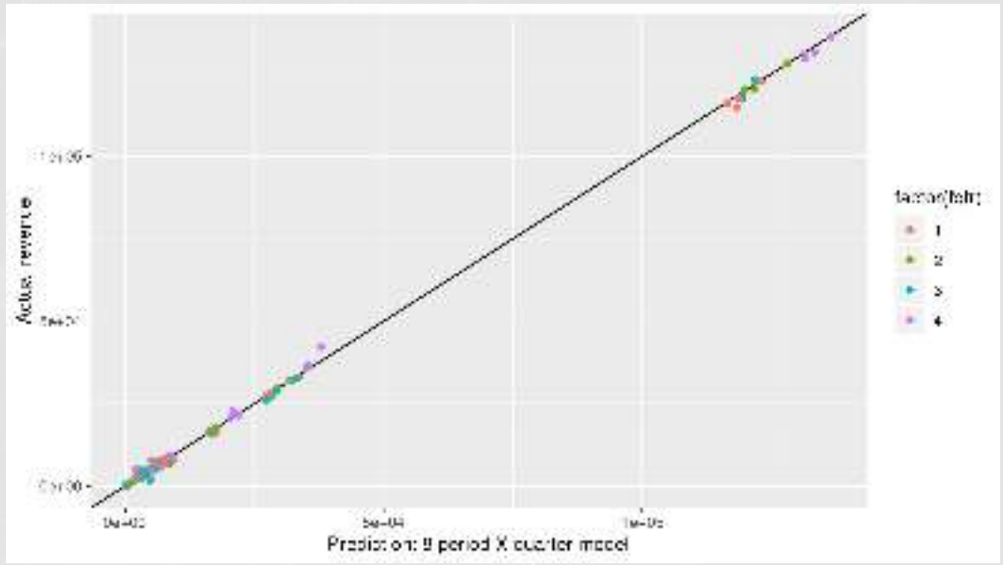
# Testing out of sample

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.9818514 | 1151.3535 | 322.73819 | 2947.3619 | 1252.5196 |
| 1 and 4 periods | 0.9954393 | 591.9500 | 156.20811 | 1400.3841 | 643.9823 |
| 8 periods | 0.9985643 | 345.8053 | 94.91083 | 677.6218 | 340.8236 |
| 8 periods w/ quarters | 0.9989231 | 298.9557 | 91.28056 | 645.5415 | 324.9395 |

## 1 quarter model




## 8 period model, by quarter

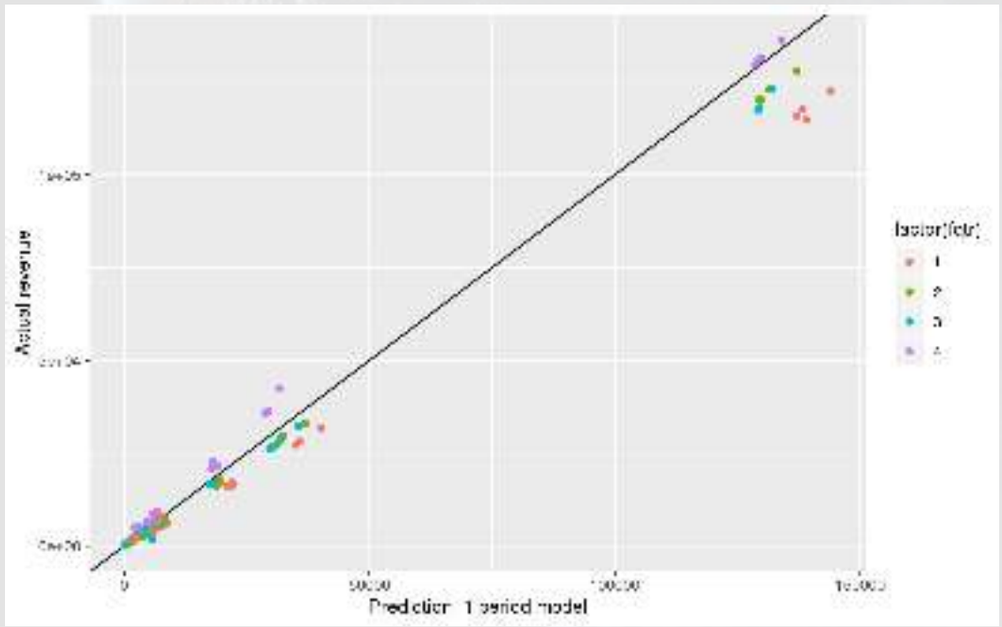


# What about for revenue growth?

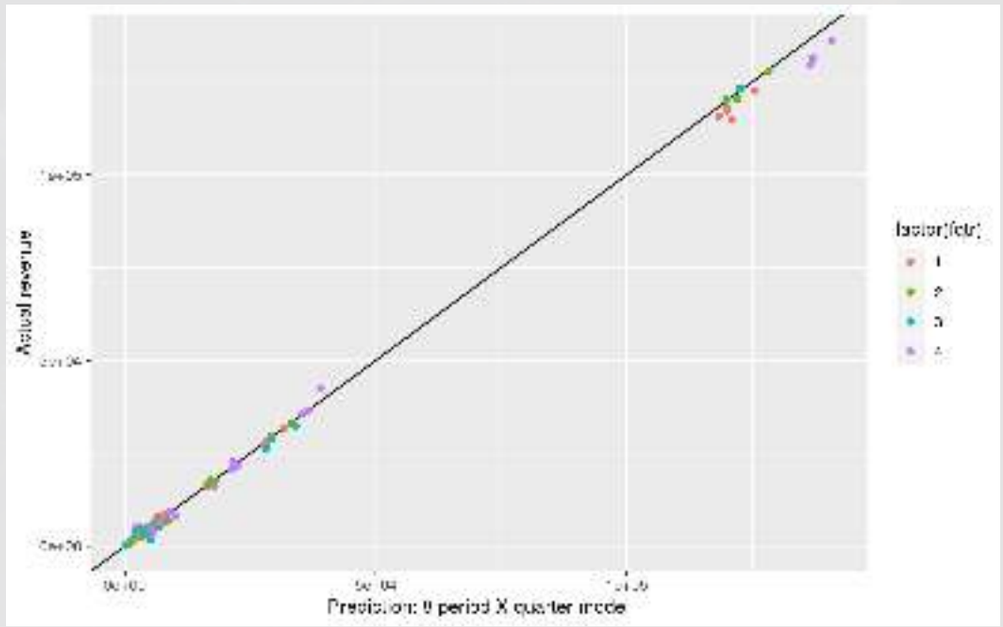
Backing out a revenue prediction, $revt_t = (1 + growth_t) \times revt_{t-1}$

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.0910390 | 1106.3730 | 308.48331 | 3374.728 | 1397.6541 |
| 1 and 4 periods | 0.4398456 | 530.6444 | 154.15086 | 1447.035 | 679.3536 |
| 8 periods | 0.6761666 | 456.2551 | 123.34075 | 1254.201 | 584.9709 |
| 8 periods w/ quarters | 0.7758834 | 378.4082 | 98.45751 | 1015.971 | 436.1522 |

## 1 quarter model




## 8 period model, by quarter



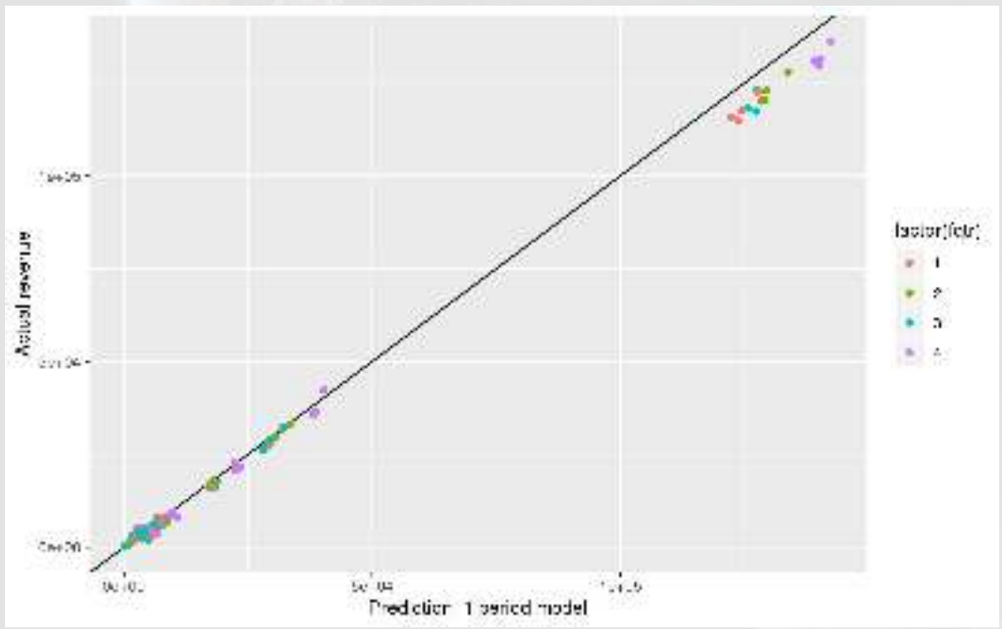
# What about for YoY revenue growth?

Backing out a revenue prediction,

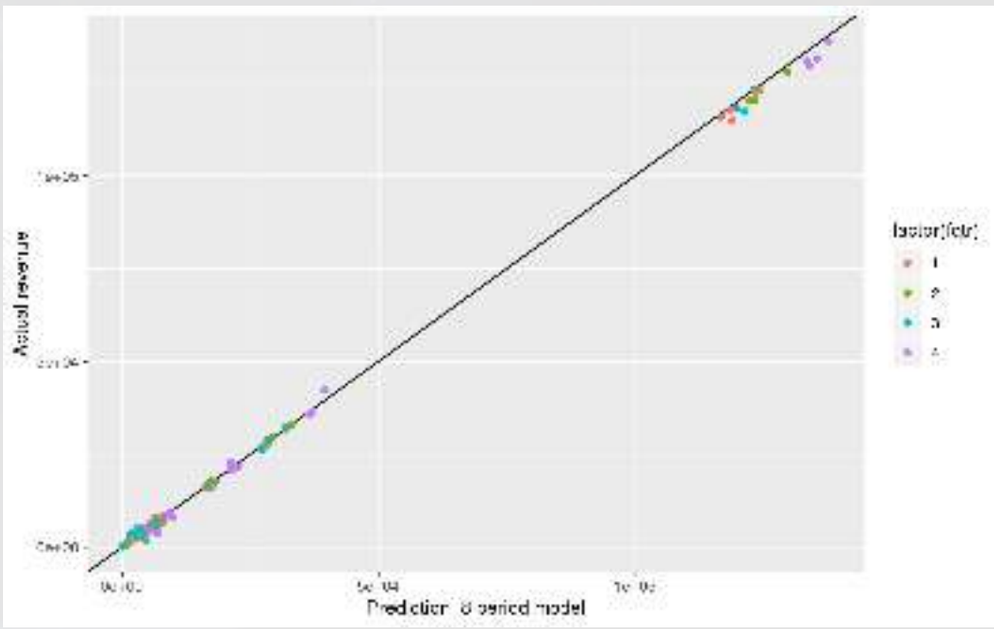
$$revt_t = (1 + yoy\_growth_t) \times revt_{t-4}$$

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.4370372 | 513.3264 | 129.2309 | 1867.4957 | 798.0327 |
| 1 and 4 periods | 0.5392281 | 487.6441 | 126.6012 | 1677.4003 | 731.2841 |
| 8 periods | 0.5398870 | 384.2923 | 101.0104 | 822.0065 | 403.5445 |
| 8 periods w/ quarters | 0.1563169 | 714.4285 | 195.3204 | 1231.8436 | 617.2989 |

## 1 quarter model




## 8 period model

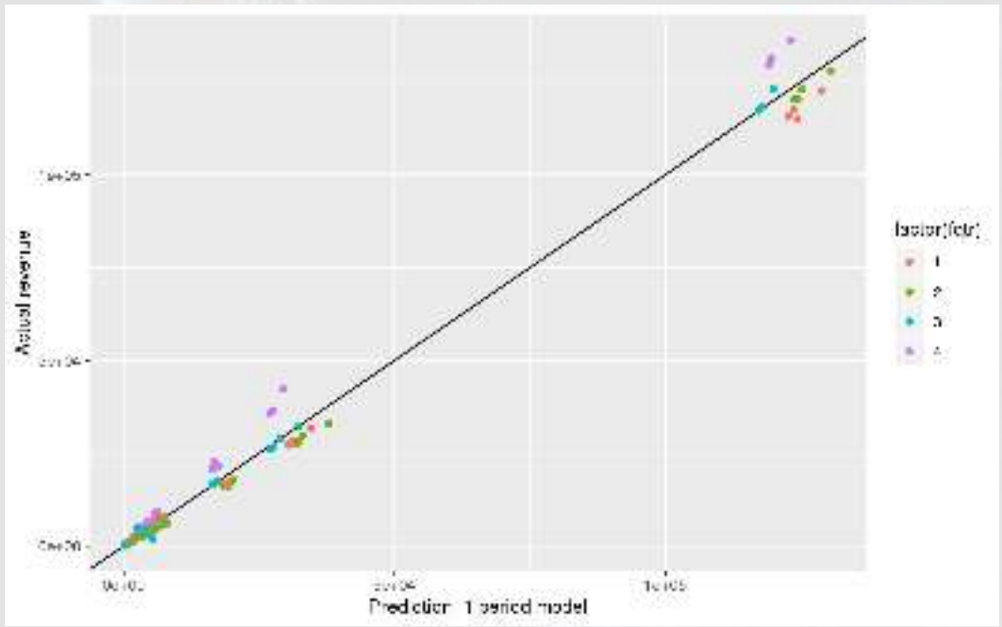


# What about for first difference?

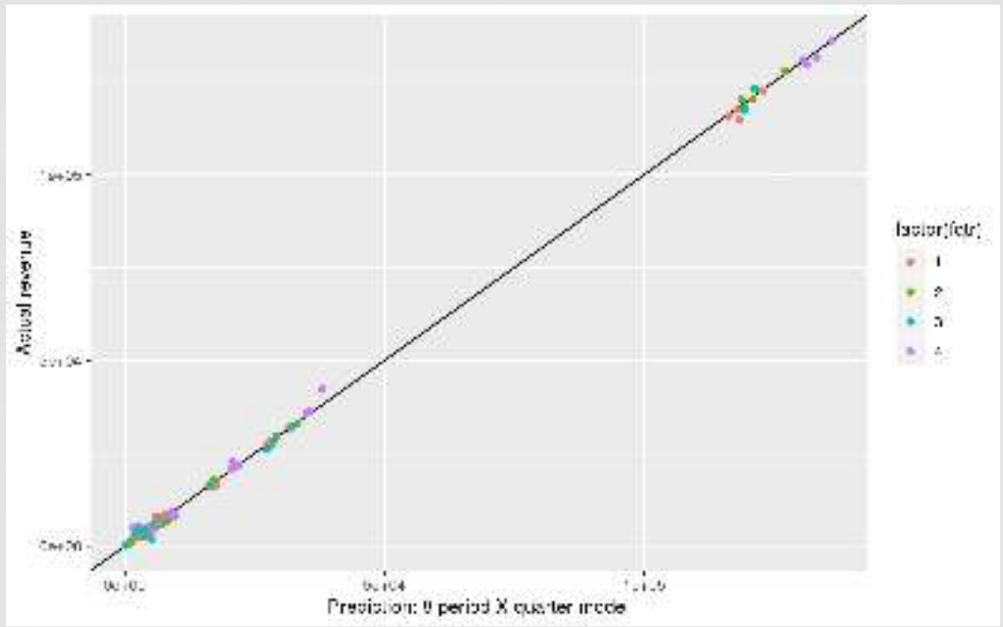
Backing out a revenue prediction, $revt_t = change_t + revt_{t-1}$

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.3532044 | 896.7969 | 287.77940 | 2252.7605 | 1022.0960 |
| 1 and 4 periods | 0.8425348 | 454.8651 | 115.52694 | 734.8120 | 377.5281 |
| 8 periods | 0.9220849 | 333.0054 | 95.95924 | 651.4967 | 320.0567 |
| 8 periods w/ quarters | 0.9397434 | 292.3102 | 86.95563 | 659.4412 | 319.7305 |

## 1 quarter model




## 8 period model, by quarter



# Takeaways

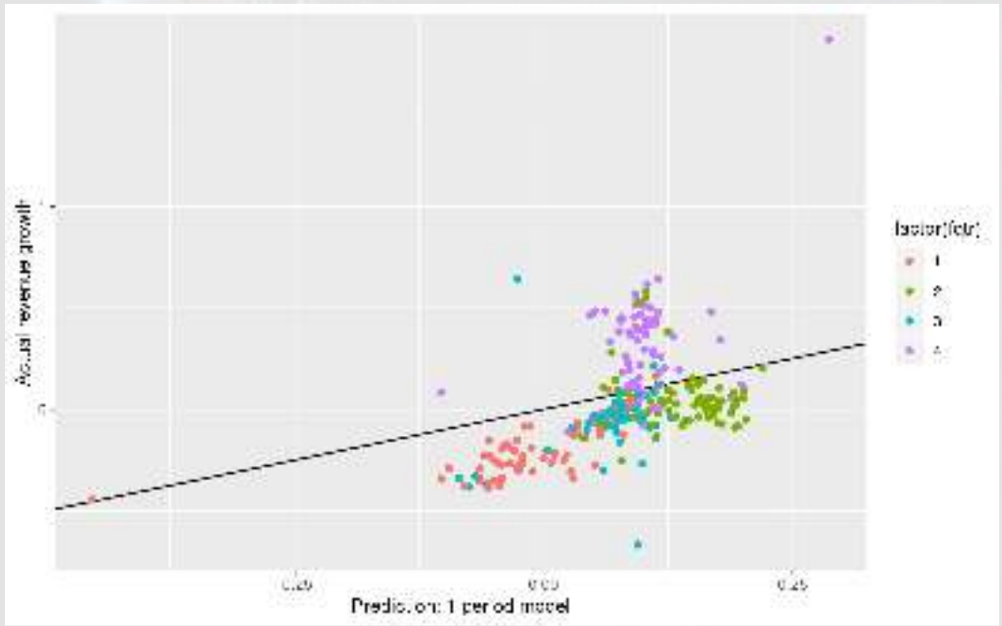
1. The first difference model works about as well as the revenue model at predicting next quarter revenue
   - From earlier, it doesn't suffer (as much) from multicollinearity either
     - This is why time series analysis is often done on first differences
       - Or second differences (difference in differences)
2. The other models perform pretty well as well
3. Extra lags generally seems helpful when accounting for cyclicality
4. Regressing by quarter helps a bit, particularly with revenue growth

# What about for revenue growth?

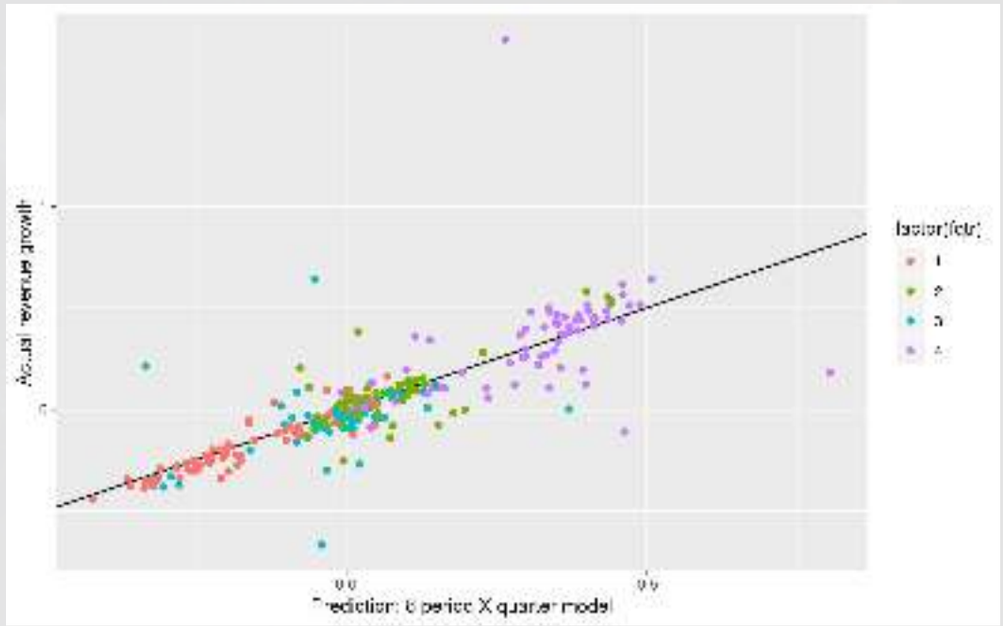
Predicting quarter over quarter revenue growth itself

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.0910390 | 0.3509269 | 0.2105219 | 0.2257396 | 0.1750580 |
| 1 and 4 periods | 0.4398456 | 0.2681899 | 0.1132003 | 0.1597771 | 0.0998087 |
| 8 periods | 0.6761666 | 0.1761825 | 0.0867347 | 0.1545298 | 0.0845826 |
| 8 periods w/ quarters | 0.7758834 | 0.1462979 | 0.0765792 | 0.1459460 | 0.0703554 |

## 1 quarter model




## 8 period model, by quarter

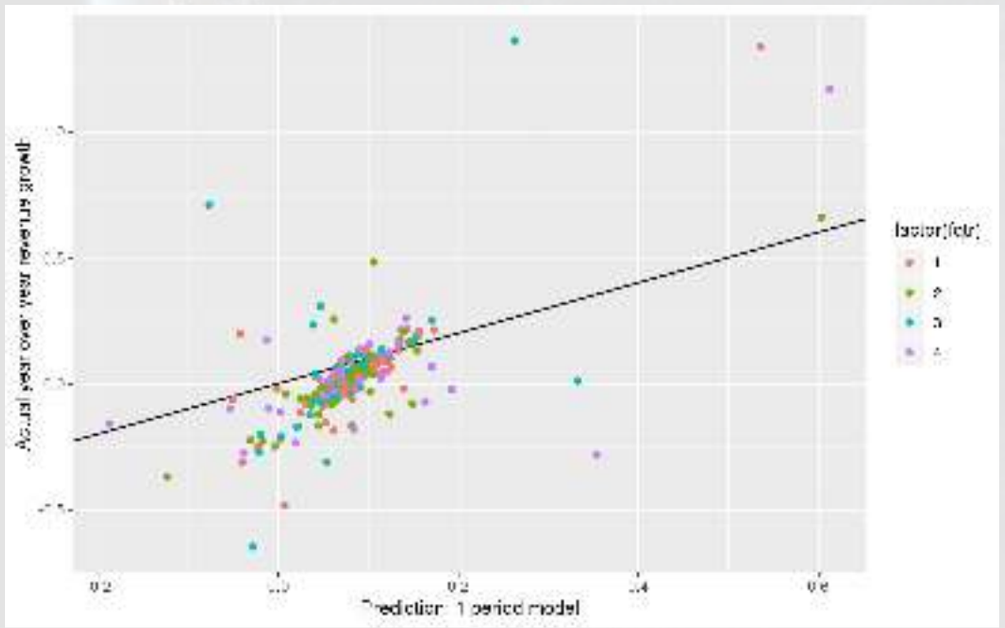


# What about for YoY revenue growth?

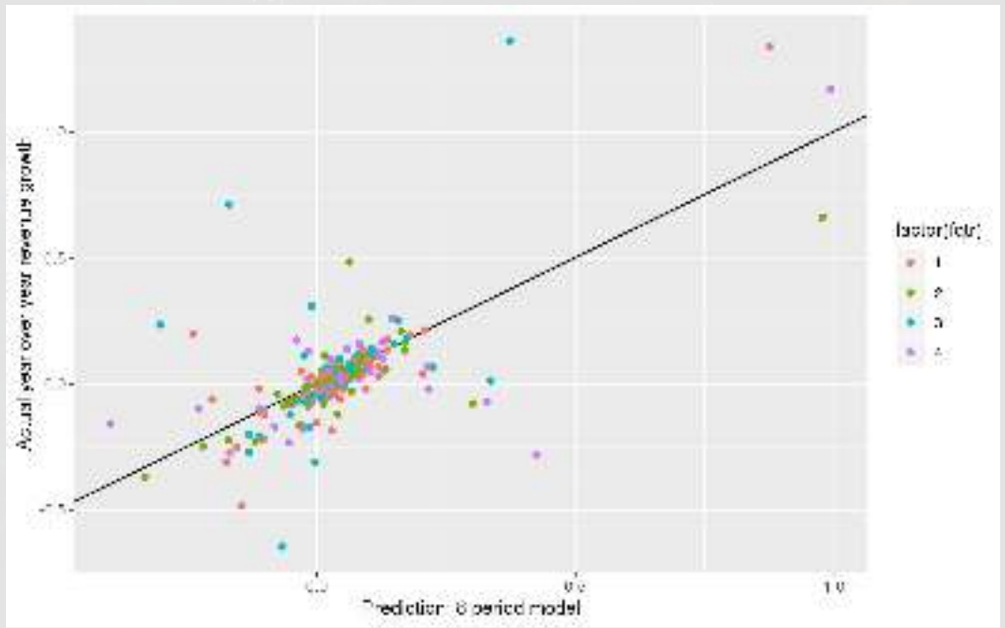
## Predicting YoY revenue growth itself

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.4370372 | 0.3116645 | 0.1114610 | 0.1515638 | 0.0942544 |
| 1 and 4 periods | 0.5392281 | 0.2451749 | 0.1015699 | 0.1498755 | 0.0896079 |
| 8 periods | 0.5398870 | 0.1928940 | 0.0764447 | 0.1346238 | 0.0658011 |
| 8 periods w/ quarters | 0.1563169 | 0.3006075 | 0.1402156 | 0.1841025 | 0.0963205 |

## 1 quarter model




## 8 period model

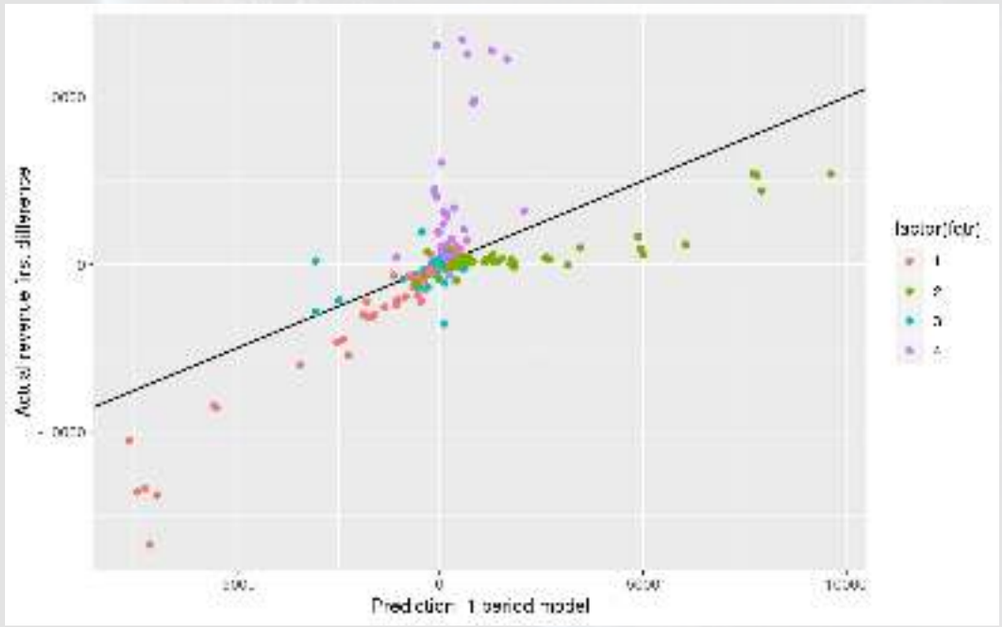


# What about for first difference?

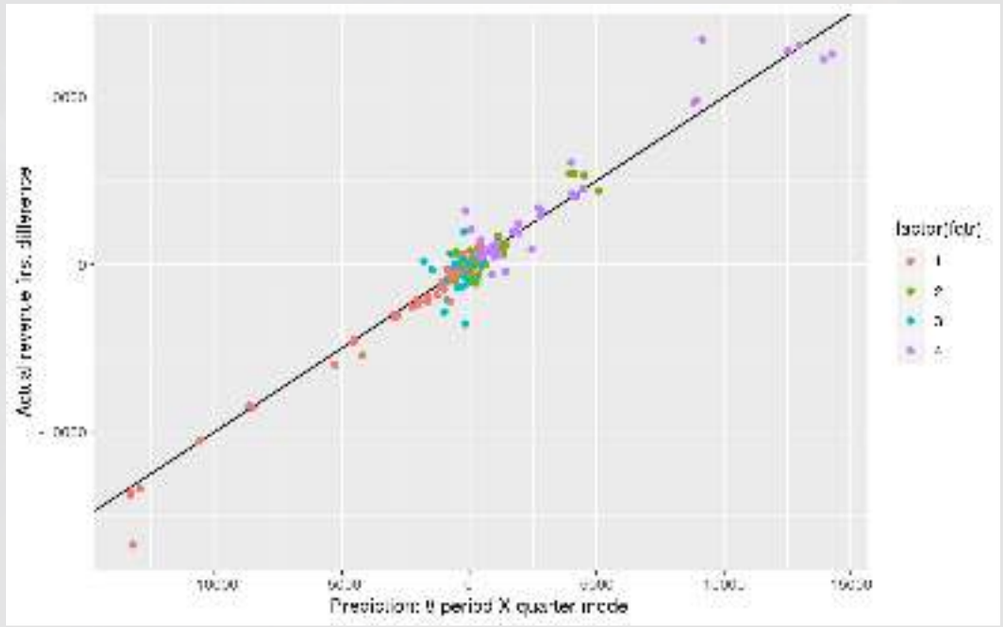
Predicting first difference in revenue itself

| | adj_r_sq | rmse_in | mae_in | rmse_out | mae_out |
|---|---|---|---|---|---|
| 1 period | 0.3532044 | 896.7969 | 287.77940 | 2252.7605 | 1022.0960 |
| 1 and 4 periods | 0.8425348 | 454.8651 | 115.52694 | 734.8120 | 377.5281 |
| 8 periods | 0.9220849 | 333.0054 | 95.95924 | 651.4967 | 320.0567 |
| 8 periods w/ quarters | 0.9397434 | 292.3102 | 86.95563 | 659.4412 | 319.7305 |

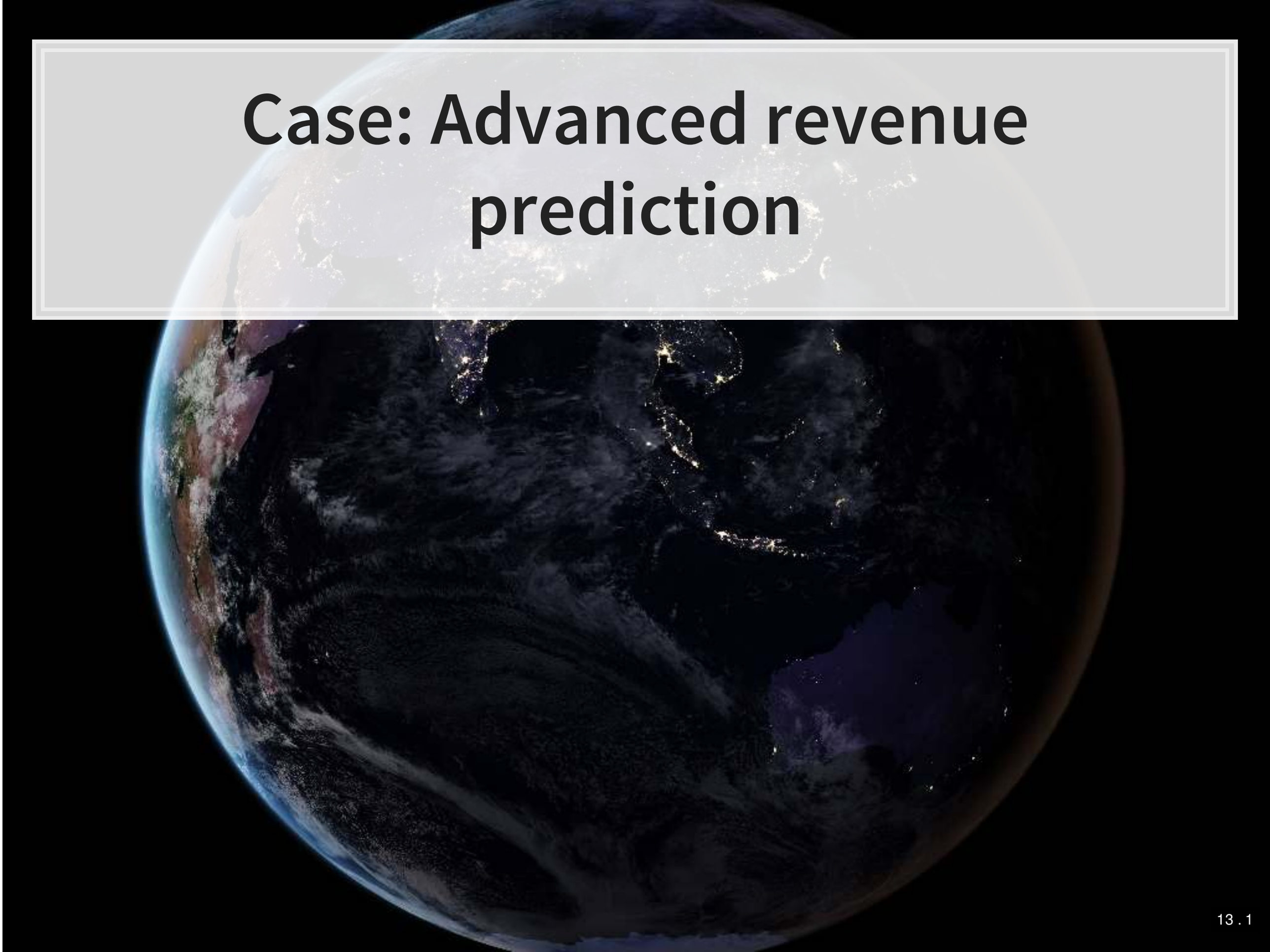
## 1 quarter model




## 8 period model, by quarter



# Case: Advanced revenue prediction



# RS Metrics' approach

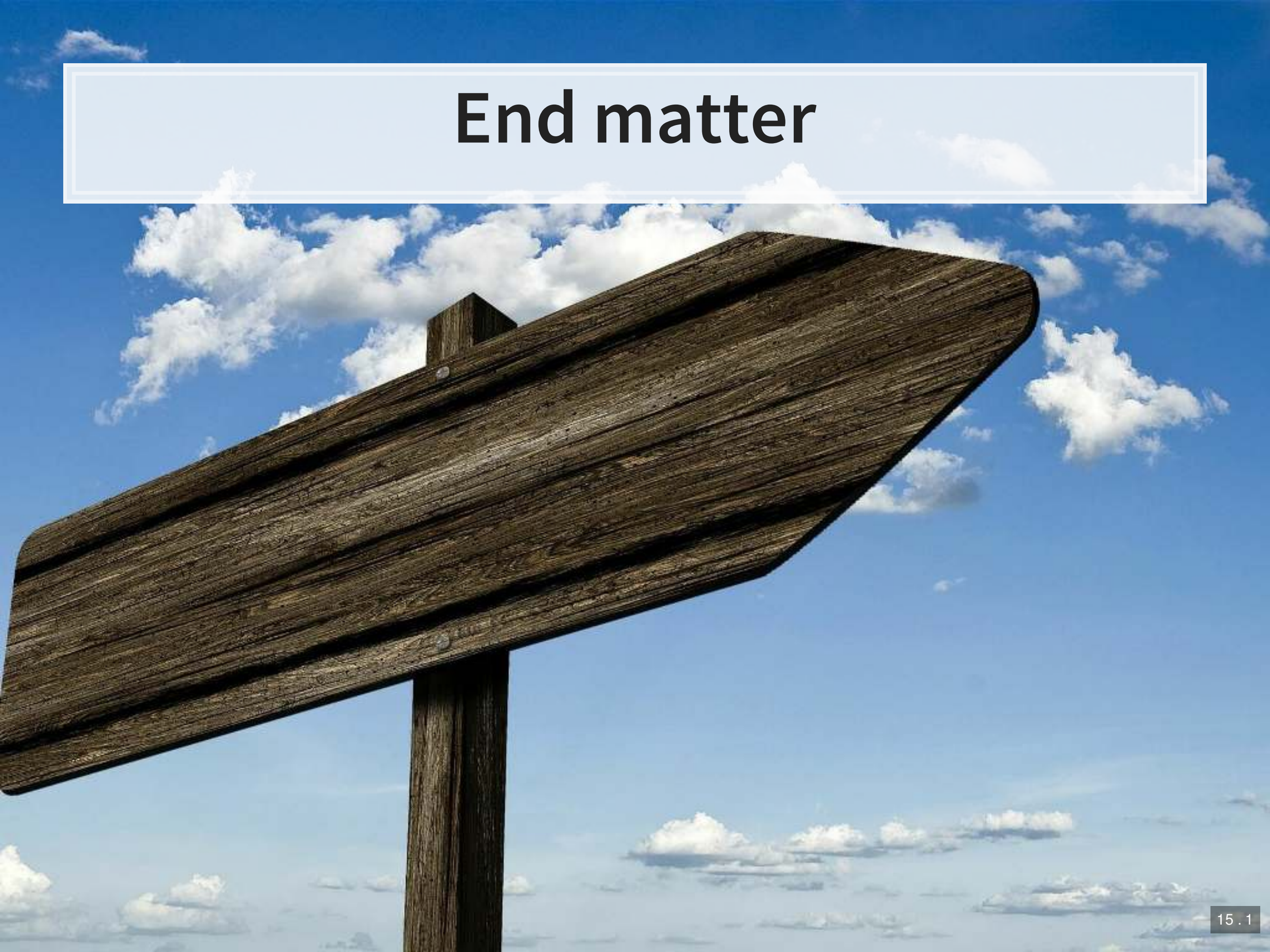
Read the press release: rmc.link/420class4

- How does RS Metrics approach revenue prediction?
- What other creative ways might there be?

# Weekly revenue prediction

# Shifted to week 5

# End matter



# For next week

- For next week:
  - First individual assignment
    - Finish by the end of Thursday
    - Submit on eLearn
  - Datacamp
    - Practice a bit more to keep up to date
      - Using R more will make it more natural

# Packages used for these slides

- kableExtra
- knitr
- lubridate
- magrittr
- revealjs
- tidyverse

# Custom code

```
# Brute force code for variable generation of quarterly data lags
df <- df %>%
  group_by(gvkey) %>%
  mutate(revtq_lag1=lag(revtq), revtq_lag2=lag(revtq, 2),
         revtq_lag3=lag(revtq, 3), revtq_lag4=lag(revtq, 4),
         revtq_lag5=lag(revtq, 5), revtq_lag6=lag(revtq, 6),
         revtq_lag7=lag(revtq, 7), revtq_lag8=lag(revtq, 8),
         revtq_lag9=lag(revtq, 9), revtq_gr=revtq / revtq_lag1 - 1,
         revtq_gr1=lag(revtq_gr), revtq_gr2=lag(revtq_gr, 2),
         revtq_gr3=lag(revtq_gr, 3), revtq_gr4=lag(revtq_gr, 4),
         revtq_gr5=lag(revtq_gr, 5), revtq_gr6=lag(revtq_gr, 6),
         revtq_gr7=lag(revtq_gr, 7), revtq_gr8=lag(revtq_gr, 8),
         revtq_yoy=revtq / revtq_lag4 - 1, revtq_yoy1=lag(revtq_yoy),
         revtq_yoy2=lag(revtq_yoy, 2), revtq_yoy3=lag(revtq_yoy, 3),
         revtq_yoy4=lag(revtq_yoy, 4), revtq_yoy5=lag(revtq_yoy, 5),
         revtq_yoy6=lag(revtq_yoy, 6), revtq_yoy7=lag(revtq_yoy, 7),
         revtq_yoy8=lag(revtq_yoy, 8), revtq_d=revtq - revtq_l1,
         revtq_d1=lag(revtq_d), revtq_d2=lag(revtq_d, 2),
         revtq_d3=lag(revtq_d, 3), revtq_d4=lag(revtq_d, 4),
         revtq_d5=lag(revtq_d, 5), revtq_d6=lag(revtq_d, 6),
         revtq_d7=lag(revtq_d, 7), revtq_d8=lag(revtq_d, 8)) %>%
  ungroup()
```

```
# Custom html table for small data frames
library(knitr)
library(kableExtra)
html_df <- function(text, cols=NULL, col1=FALSE, full=F) {
  if(!length(cols)) {
    cols=colnames(text)
  }
  if(!col1) {
    kable(text,"html", col.names = cols, align = c("l",rep('c',length(cols)-1))) %>%
      kable_styling(bootstrap_options = c("striped","hover"), full_width=full)
  } else {
    kable(text,"html", col.names = cols, align = c("l",rep('c',length(cols)-1))) %>%
      kable_styling(bootstrap_options = c("striped","hover"), full_width=full) %>%
      column_spec(1,bold=T)
  }
}
```

# Custom code

```
# These functions are a bit ugly, but can construct many charts quickly
# eval(parse(text=var)) is just a way to convert the string name to a variable reference
# Density plot for 1st to 99th percentile of data
plt_dist <- function(df,var) {
  df %>%
    filter(eval(parse(text=var)) < quantile(eval(parse(text=var)),0.99, na.rm=TRUE),
           eval(parse(text=var)) > quantile(eval(parse(text=var)),0.01, na.rm=TRUE)) %>%
    ggplot(aes(x=eval(parse(text=var)))) +
    geom_density() + xlab(var)
}
```

```
# Density plot for 1st to 99th percentile of both columns
plt_bar <- function(df,var) {
  df %>%
    filter(eval(parse(text=var)) < quantile(eval(parse(text=var)),0.99, na.rm=TRUE),
           eval(parse(text=var)) > quantile(eval(parse(text=var)),0.01, na.rm=TRUE)) %>%
    ggplot(aes(y=eval(parse(text=var)), x=fqtr)) +
    geom_bar(stat = "summary", fun.y = "mean") + xlab(var)
}
```

```
# Scatter plot with lag for 1st to 99th percentile of data
plt_sct <- function(df,var1, var2) {
  df %>%
    filter(eval(parse(text=var1)) < quantile(eval(parse(text=var1)),0.99, na.rm=TRUE),
           eval(parse(text=var2)) < quantile(eval(parse(text=var2)),0.99, na.rm=TRUE),
           eval(parse(text=var1)) > quantile(eval(parse(text=var1)),0.01, na.rm=TRUE),
           eval(parse(text=var2)) > quantile(eval(parse(text=var2)),0.01, na.rm=TRUE)) %>%
    ggplot(aes(y=eval(parse(text=var1)), x=eval(parse(text=var2)), color=factor(fqtr))) +
    geom_point() + geom_smooth(method = "lm") + ylab(var1) + xlab(var2)
}
```

```
# Calculating various in and out of sample statistics
models <- list(mod1,mod2,mod3,mod4)
model_names <- c("1 period", "1 and 4 period", "8 periods", "8 periods w/ quarters")

df_test <- data.frame(adj_r_sq=sapply(models, function(x)summary(x)[["adj.r.squared"]]),
                      rmse_in=sapply(models, function(x)rmse(train$revtq, predict(x,train))),
                      mae_in=sapply(models, function(x)mae(train$revtq, predict(x,train))),
                      rmse_out=sapply(models, function(x)rmse(test$revtq, predict(x,test))),
                      mae_out=sapply(models, function(x)mae(test$revtq, predict(x,test))))
rownames(df_test) <- model_names
html_df(df_test)  # Custom function using knitr and kableExtra
```