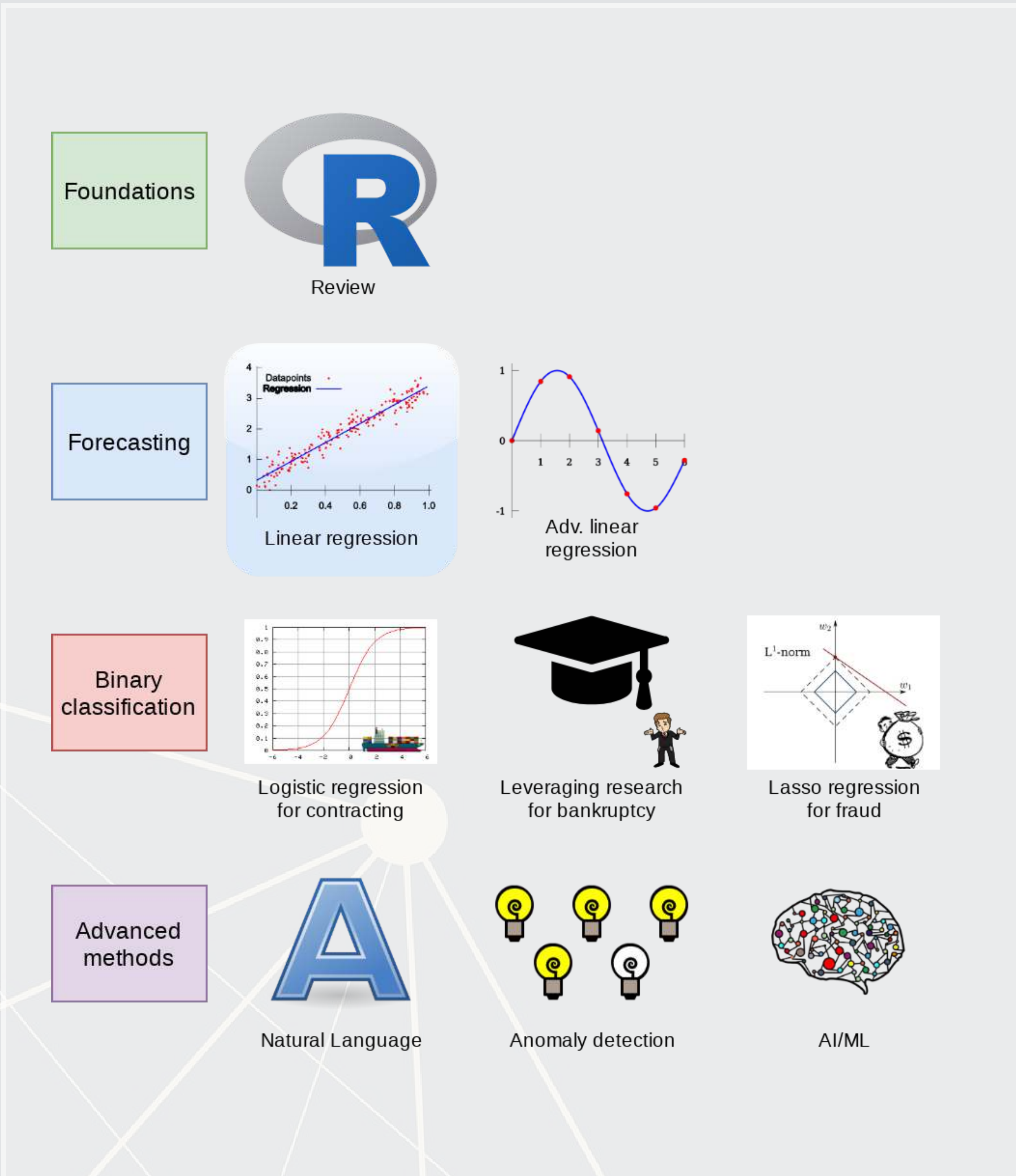
ACCT 420: Linear Regression

Session 2

Dr. Richard M. Crowley

Front matter

Learning objectives



- **Theory:**
  - Develop a logical approach to problem solving with data
    - Statistics
    - Causation
    - Hypothesis testing
- **Application:**
  - Predicting revenue for real estate firms
- **Methodology:**
  - Univariate stats
  - Linear regression
  - Visualization

Datacamp

- For next week:
  - Just 2 chapters:
    - 1 on linear regression
    - 1 on tidyverse methods
- The full list of Datacamp materials for the course is up on eLearn

R Installation

- If you haven't already, make sure to install R and R Studio!
  - Instructions are in Session 1's slides
  - You will need it for this week's assignment
- Please install a few packages using the following code
  - These packages are also needed for the first assignment
  - You are welcome to explore other packages as well, but those will not be necessary for now

```
# Run this in the R Console inside RStudio
install.packages(c("tidyverse","plotly","tufte","reshape2"))
```

- Assignments will be provided as R Markdown files

> The format will generally all be filled out – you will just add to it, answer questions, analyze data, and explain your work. Instructions and hints are in the same file

R Markdown: A quick guide

- Headers and subheaders start with `#` and `##`, respectively
- Code blocks starts with `` ```{r} `` and end with `` ``` ``
  - By default, all code and figures will show up in the document
- Inline code goes in a block starting with `` `r  ` `` and ending with `` ` ``
- Italic font can be used by putting `*` or `_` around text
- Bold font can be used by putting `**` around text
  - E.g.: `**bold text**` becomes **bold text**
- To render the document, click Knit
- Math can be placed between `$` to use LaTeX notation
  - E.g. `$\frac{revt}{at}$` becomes $\frac{revt}{at}$
- Full equations (on their own line) can be placed between `$$`
- A block quote is prefixed with `>`
- For a complete guide, see R Studio's R Markdown::Cheat Sheet

Application: Revenue prediction

The question

> How can we predict revenue for a company, leveraging data about that company, related companies, and macro factors

- Specific application: Real estate companies

More specifically…

- Can we use a company’s own accounting data to predict it’s future revenue?
- Can we use other companies’ accounting data to better predict all of their future revenue?
- Can we augment this data with macro economic data to further improve prediction?
  - Singapore business sentiment data

Linear models

What is a linear model?

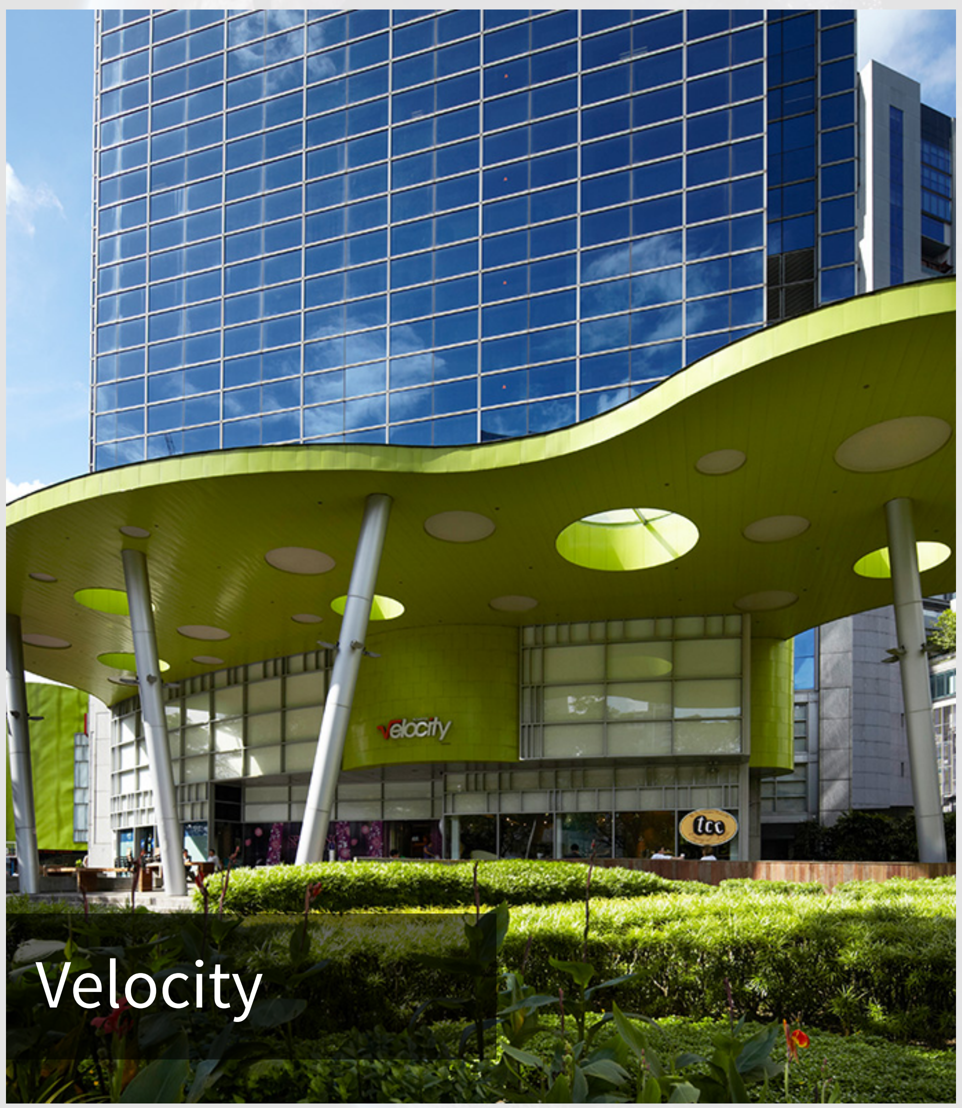
$$\hat{y} = \alpha + \beta \hat{x} + \varepsilon$$

- The simplest model is trying to predict some outcome $\hat{y}$ as a function of an input $\hat{x}$
  - $\hat{y}$ in our case is a firm's revenue in a given year
  - $\hat{x}$ could be a firm's assets in a given year
  - α and β are solved for
  - ε is the error in the measurement

> I will refer to this as an *OLS* model – **O**rdinary **L**east **S**quare regression

Example

Let's predict UOL's revenue for 2016



- Compustat has data for them since 1989
  - Complete since 1994
    - Missing CapEx before that

```
# revt: Revenue, at: Assets
summary(uol[,c("revt", "at")])
```

```
##       revt                at       
##  Min.   :  94.78   Min.   : 1218  
##  1st Qu.: 193.41   1st Qu.: 3044  
##  Median : 427.44   Median : 3478  
##  Mean   : 666.38   Mean   : 5534  
##  3rd Qu.:1058.61   3rd Qu.: 7939  
##  Max.   :2103.15   Max.   :19623  
```

Linear models in R

- To run a linear model, use `lm()`
  - The first argument is a formula for your model, where `~` is used in place of an equals sign
    - The left side is what you want to predict
    - The right side is inputs for prediction, separated by `+`
  - The second argument is the data to use
- Additional variations for the formula:
  - Functions transforming inputs (as vectors), such as `log()`
  - Fully interacting variables using `*`
    - I.e., `A*B` includes, A, B, and A times B in the model
  - Interactions using `:`
    - I.e., `A:B` just includes A times B in the model

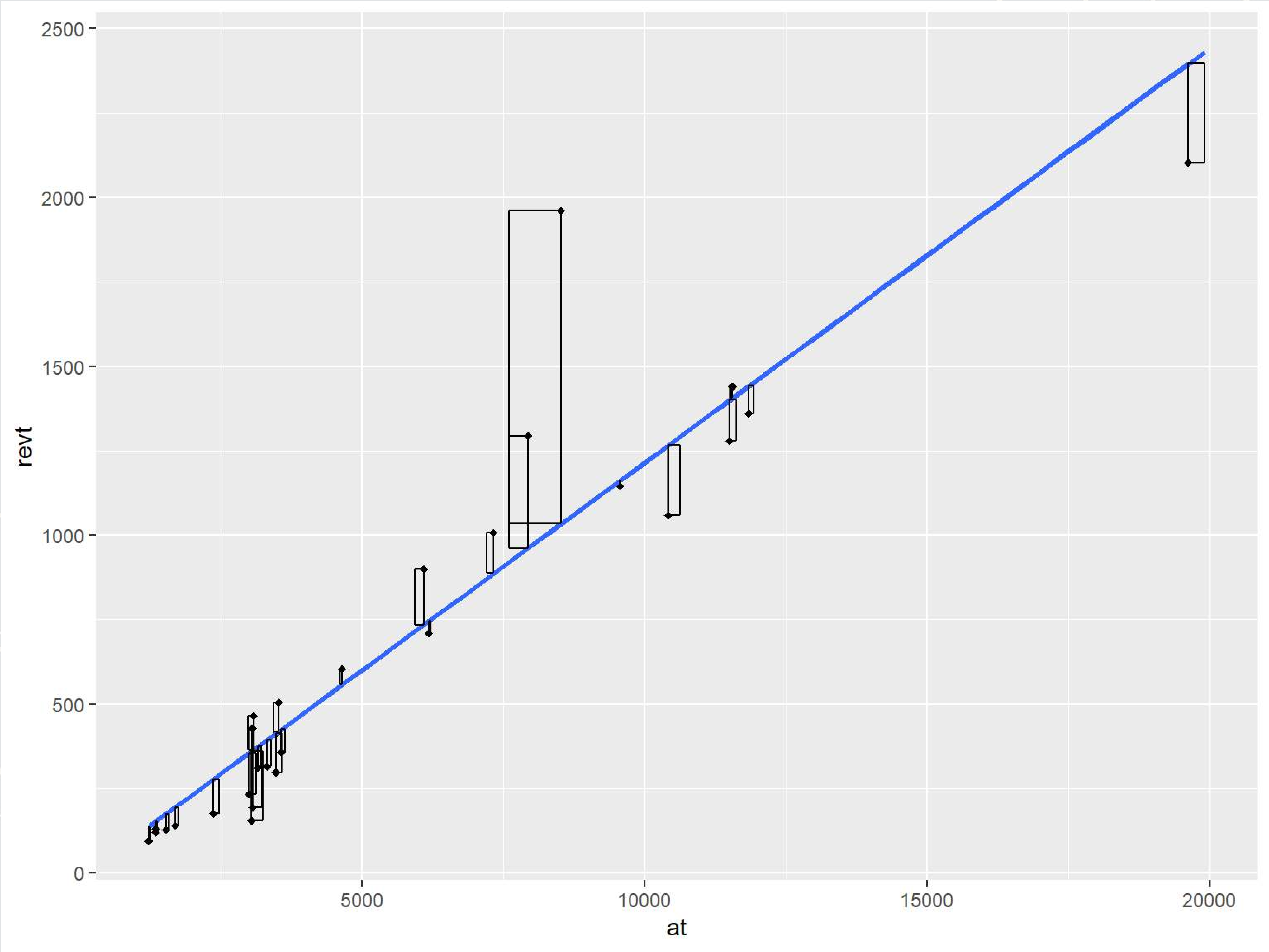
```
# Example:
lm(revt ~ at, data = uol)
```

Example: UOL

```r
mod1 <- lm(revt ~ at, data = uol)
summary(mod1)
```

```
##
## Call:
## lm(formula = revt ~ at, data = uol)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -295.01 -101.29  -41.09   47.17  926.29
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.831399  67.491305  -0.205    0.839
## at            0.122914   0.009678  12.701  6.7e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 221.2 on 27 degrees of freedom
## Multiple R-squared:  0.8566, Adjusted R-squared:  0.8513
## F-statistic: 161.3 on 1 and 27 DF,  p-value: 6.699e-13
```

Why is it called Ordinary Least Squares?



Example: UOL

- This model wasn't so interesting…
  - Bigger firms have more revenue – this is a given
- How about… revenue *growth*?
- And *change* in assets
  - i.e., Asset growth

$$\Delta x_t = \frac{x_t}{x_{t-1}} - 1$$

Calculating changes in R

- The easiest way is using `tidyverse`'s `dplyr`
  - This has a `lag()` function
- The default way to do it is to create a vector manually

```
# tidyverse
uol <- uol %>%
  mutate(revt_growth1 = revt / lag(revt) - 1)

# R way
uol$revt_growth2 = uol$revt / c(NA, uol$revt[-length(uol$revt)]) - 1

identical(uol$revt_growth1, uol$revt_growth2)
```

```
## [1] TRUE
```

> You can use whichever you are comfortable with

A note on mutate()

- `mutate()` adds variables to an existing data frame
  - Also `mutate_all()`, `mutate_at()`, `mutate_if()`
  - `mutate_all()` applies a transformation to all values in a data frame and adds these to the data frame
  - `mutate_at()` does this for a set of specified variables
  - `mutate_if()` transforms all variables matching a condition
    - Such as `is.numeric`
- Mutate can be very powerful when making more complex variables
  - For instance: Calculating growth within company in a multi-company data frame
  - It's way more than needed for a simple ROA though.

Example: UOL with changes

```r
# Make the other needed change
uol <- uol %>%
  mutate(at_growth = at / lag(at) - 1) %>%  # Calculate asset growth
  rename(revt_growth = revt_growth1)         # Rename for readability
# Run the OLS model
mod2 <- lm(revt_growth ~ at_growth, data = uol)
summary(mod2)
```

```
##
## Call:
## lm(formula = revt_growth ~ at_growth, data = uol)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.57736 -0.10534 -0.00953  0.15132  0.42284
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.09024    0.05620   1.606   0.1204
## at_growth    0.53821    0.27717   1.942   0.0631 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2444 on 26 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.1267, Adjusted R-squared:  0.09307
## F-statistic: 3.771 on 1 and 26 DF,  p-value: 0.06307
```

Example: UOL with changes

- ΔAssets doesn't capture ΔRevenue so well
- Perhaps change in total assets is a bad choice?
- Or perhaps we need to expand our model?

Scaling up!

$$\hat{y} = \alpha + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \ldots + \varepsilon$$

- OLS doesn't need to be restricted to just 1 input!
  - Not unlimited though (yet)
    - Number of inputs must be less than the number of observations minus 1
- Each $\hat{x}_i$ is an input in our model
- Each β_i is something we will solve for
- $\hat{y}$, α, and ε are the same as before

Scaling up our model

We have… 464 variables from Compustat Global alone!

- Let's just add them all?

- We only have 28 observations…
  - 28 << 464…

Now what?

Scaling up our model

> Building a model requires careful thought!

- This is where having accounting and business knowledge comes in!

> What makes sense to add to our model?

Practice: mutate()

- This practice is to make sure you understand how to use mutate with leads and lags
  - These are very important when dealing with business data!
- Do exercises 1 and 2 on today's R practice file:
  - R Practice
  - Shortlink: rmc.link/420r2

Statistics Foundations

Frequentist statistics

> A specific test is one of an infinite number of replications

- The “correct” answer should occur most frequently, i.e., with a high probability
- Focus on true vs false
- Treat unknowns as fixed constants to figure out
  - Not random quantities
- Where it’s used
  - Classical statistics methods
    - Like OLS

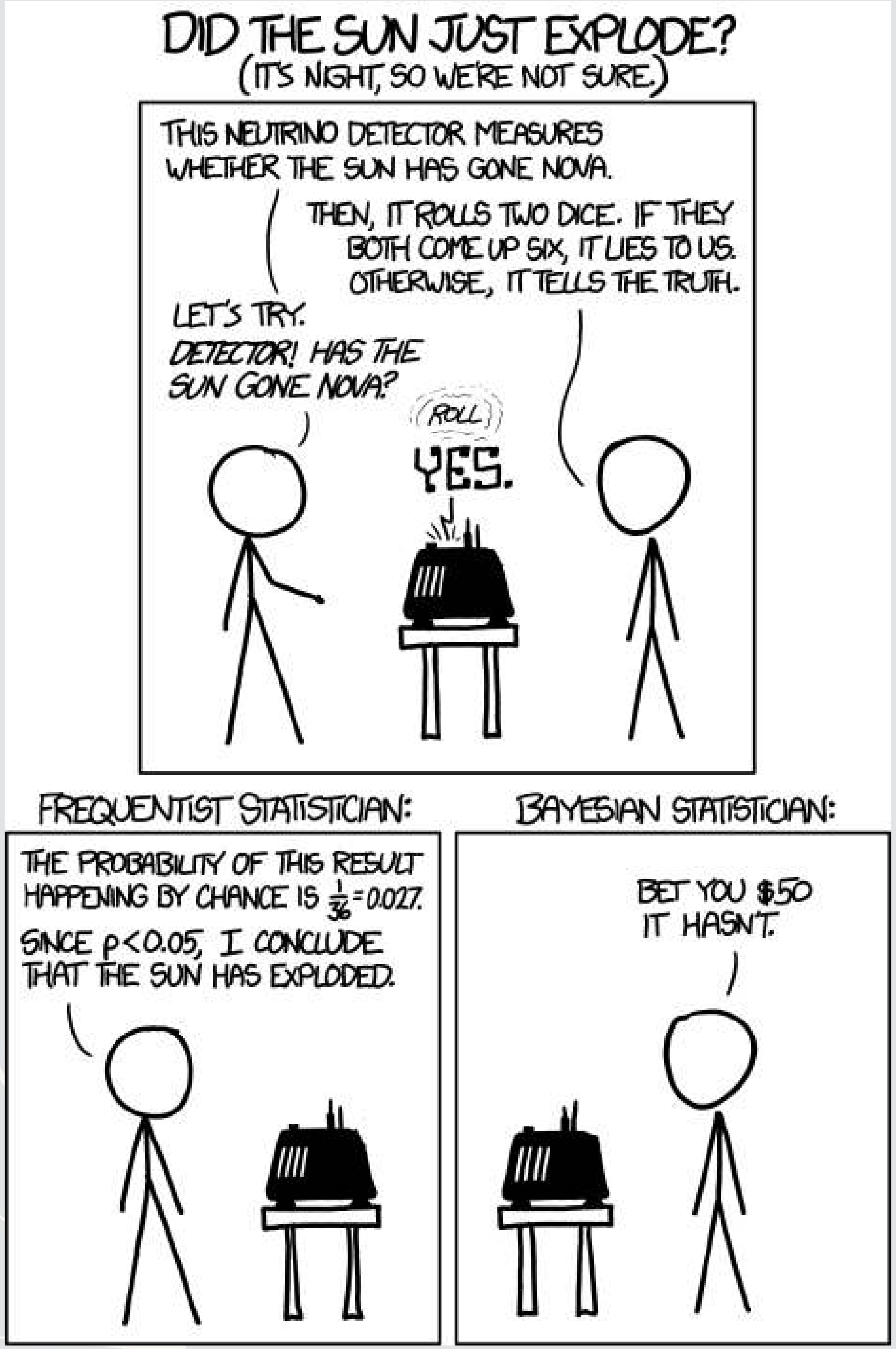
Bayesian statistics

Focus on distributions and beliefs

- Prior distribution – what is believed before the experiment
- Posterior distribution: an updated belief of the distribution due to the experiment
- Derive distributions of parameters
- Where it’s used:
  - Many machine learning methods
    - Bayesian updating acts as the learning
  - Bayesian statistics

A separate school of statistics thought

Frequentist vs Bayesian methods



Frequentist perspective: Repeat the test

```r
detector <- function() {
  dice <- sample(1:6, size=2, replace=TRUE)
  if (sum(dice) == 12) {
    "exploded"
  } else {
    "still there"
  }
}

experiment <- replicate(1000,detector())
# p value
p <- sum(experiment == "still there") / 1000
if (p < 0.05) {
  paste("p-value: ", p, "-- Fail to reject H_A, sun appears to have exploded")
} else {
  paste("p-value: ", p, "-- Reject H_A that sun exploded")
}
```

```
## [1] "p-value:  0.97 -- Reject H_A that sun exploded"
```

> Frequentist: The sun didn't explode

Bayes persepctive: Bayes rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- A: The sun exploded
- B: The detector said it exploded
- $P(A)$: Really, really small. Say, ~0.
- $P(B)$: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
- $P(B|A)$: $\frac{35}{36}$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{\frac{35}{36} \times \sim 0}{\frac{1}{36}} = 35 \times \sim 0 \approx 0$$

> Bayesian: The sun didn’t explode

What analytics typically relies on

- Regression approaches
  - Most often done in a frequentist manner
  - Can be done in a Bayesian manner as well
- Artificial Intelligence
  - Often frequentist
  - Sometimes neither – “It just works”
- Machine learning
  - Sometimes Bayesian, sometime frequentist
  - We’ll see both

> We will use both to some extent – for our purposes, we will not debate the merits of either school of thought, but use tools derived from both

Confusion from frequentist approaches

- Possible contradictions:
  - F test says the model is good yet nothing is statistically significant
  - Individual p-values are good yet the model isn't
  - One measure says the model is good yet another doesn't

> There are many ways to measure a model, each with their own merits. They don't always agree, and it's on us to pick a reasonable measure.

Formalizing frequentist testing

Why formalize?

- Our current approach has been ad hoc
  - What is our goal?
  - How will we know if we have achieved it?
- Formalization provides more rigor

Scientific method

1. Question
   - What are we trying to determine?
2. Hypothesis
   - What do we think will happen? Build a model
3. Prediction
   - What exactly will we test? Formalize model into a statistical approach
4. Testing
   - Test the model
5. Analysis
   - Did it work?

Hypotheses

- Null hypothesis, a.k.a. H_0
  - The status quo
  - Typically: The model *doesn't* work
- Alternative hypothesis, a.k.a. H_1 or H_A
  - The model *does* work (and perhaps how it works)
- Frequentist statistics can never directly support H_0!
  - Only can fail to find support for H_A
  - Even if our p-value is 1, we can't say that the results prove the null hypothesis!

> We will use test statistics to test the hypotheses

Regression

- Regression (like OLS) has the following assumptions
  1. The data is generated following some model
     - E.g., a linear model
       - Next week, a logistic model
  2. The data conforms to some statistical properties as required by the test
  3. The model coefficients are something to precisely determine
     - I.e., the coefficients are constants
  4. p-values provide a measure of the chance of an error in a particular aspect of the model
     - For instance, the p-value on β_1 in $y = \alpha + \beta_1 x_1 + \varepsilon$ essentially gives the probability that the sign of β_1 is wrong

OLS Statistical properties

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \varepsilon$$

$$\hat{y} = \alpha + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \ldots + \hat{\varepsilon}$$

1. There should be a *linear* relationship between y and each x_i
   - I.e., y is [approximated by] a constant multiple of each x_i
   - Otherwise we **shouldn't** use a *linear* regression
2. Each $\hat{x}_i$ is normally distributed
   - Not so important with larger data sets, but a good to adhere to
3. Each observation is independent
   - We'll violate this one for the sake of *causality*
4. Homoskedasticity: Variance in errors is constant
   - This is important
5. Not too much multicollinearity
   - Each $\hat{x}_i$ should be relatively independent from the others
   - Some is OK

Practical implications

> Models designed under a frequentist approach can only answer the question of "does this matter?"

- Is this a problem?

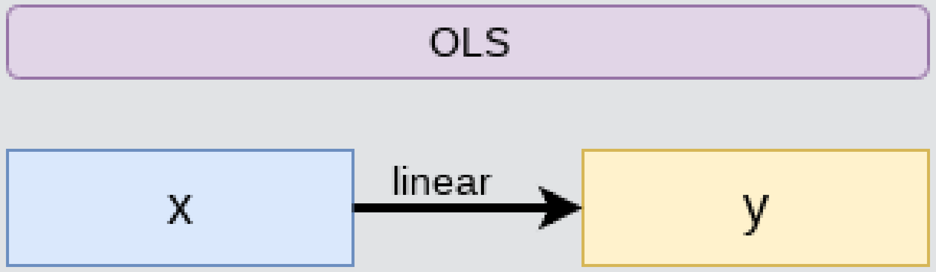
Linear model implementation

What exactly is a linear model?

- Anything OLS is linear
- Many transformations can be recast to linear
  - Ex.: $log(y) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 {x_1}^2 + \beta_4 x_1 \cdot x_2$
    - This is the same as $y' = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ where:
      - $y' = log(y)$
      - $x_3 = {x_1}^2$
      - $x_4 = x_1 \cdot x_2$

> Linear models are *very* flexible

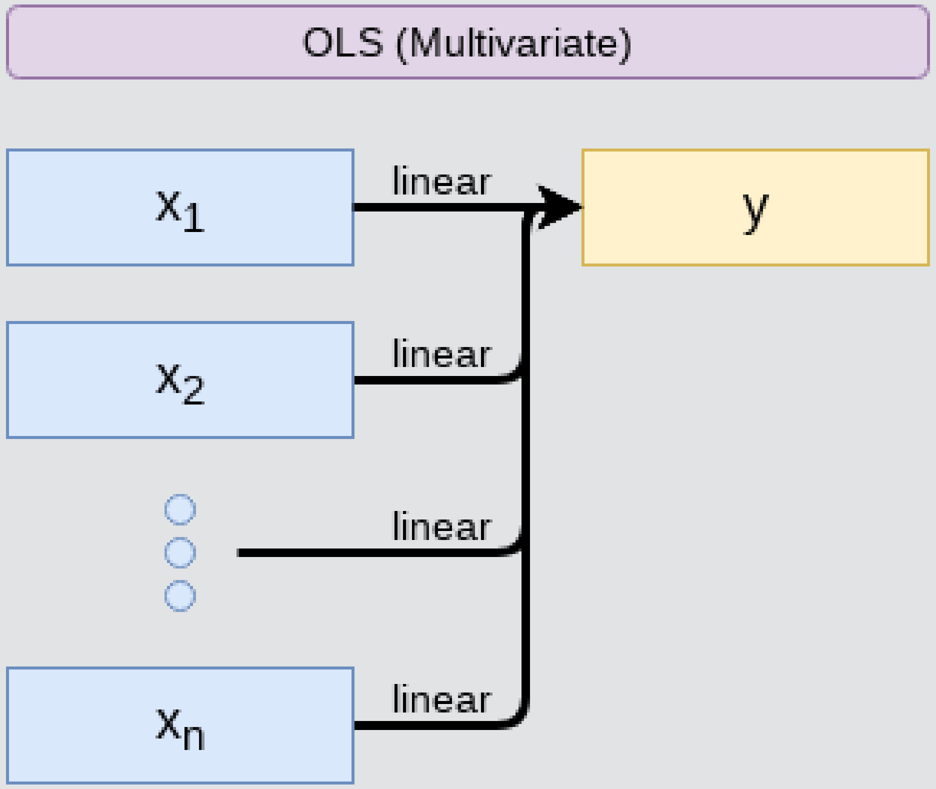
Mental model of OLS: 1 input



> Simple OLS measures a simple linear relationship between an input and an output

- E.g.: Our first regression this week: Revenue on assets

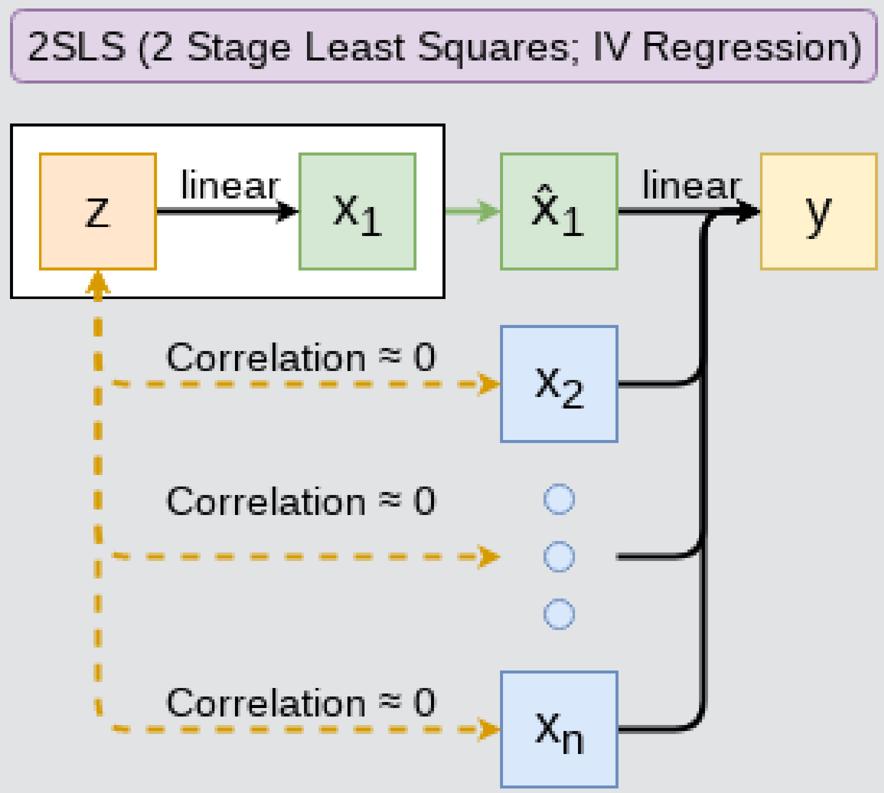
Mental model of OLS: Multiple inputs



> OLS measures simple linear relationships between a set of inputs and one output

- E.g.: This is what we did when scaling up earlier this session

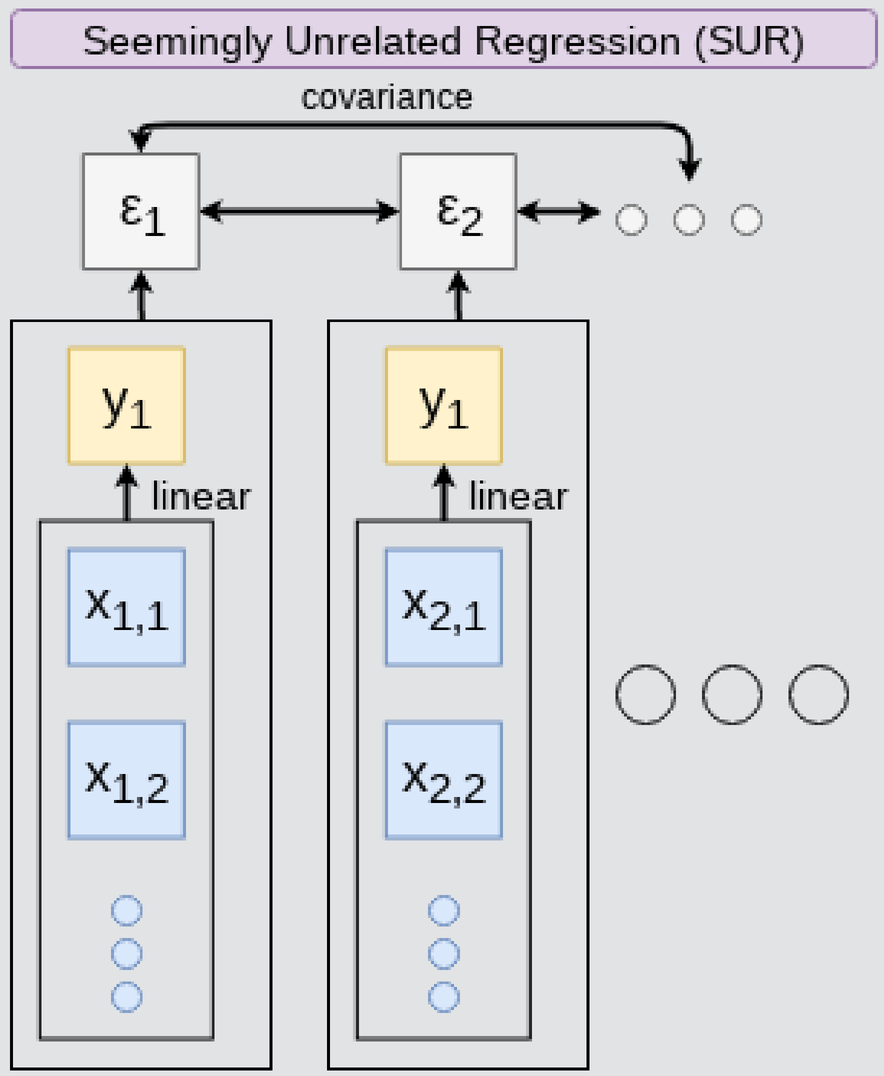
Other linear models: IV Regression (2SLS)



> IV/2SLS models linear relationships where the effect of some x_i on y may be confounded by outside factors.

- E.g.: Modeling the effect of management pay duration (like bond duration) on firms' choice to issue earnings forecasts
  - Instrument with CEO tenure (Cheng, Cho, and Kim 2015)

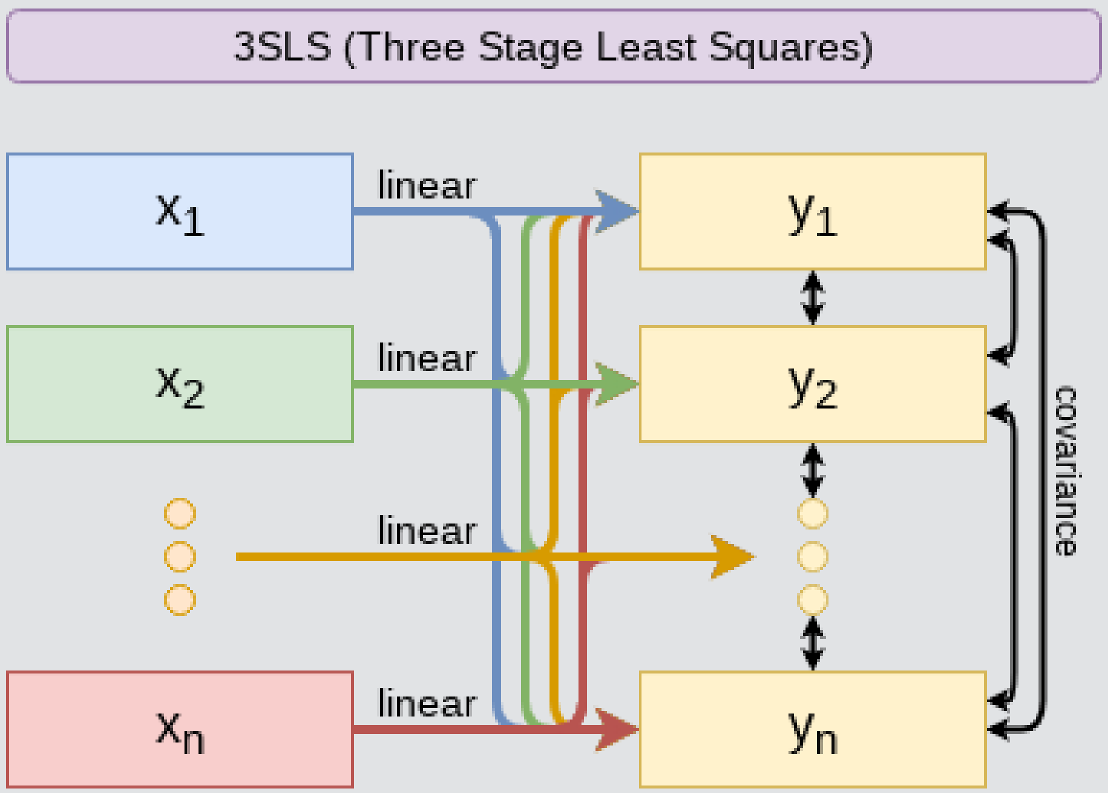
Other linear models: SUR



SUR models systems with related error terms

- E.g.: Modeling both revenue and earnings simultaneously

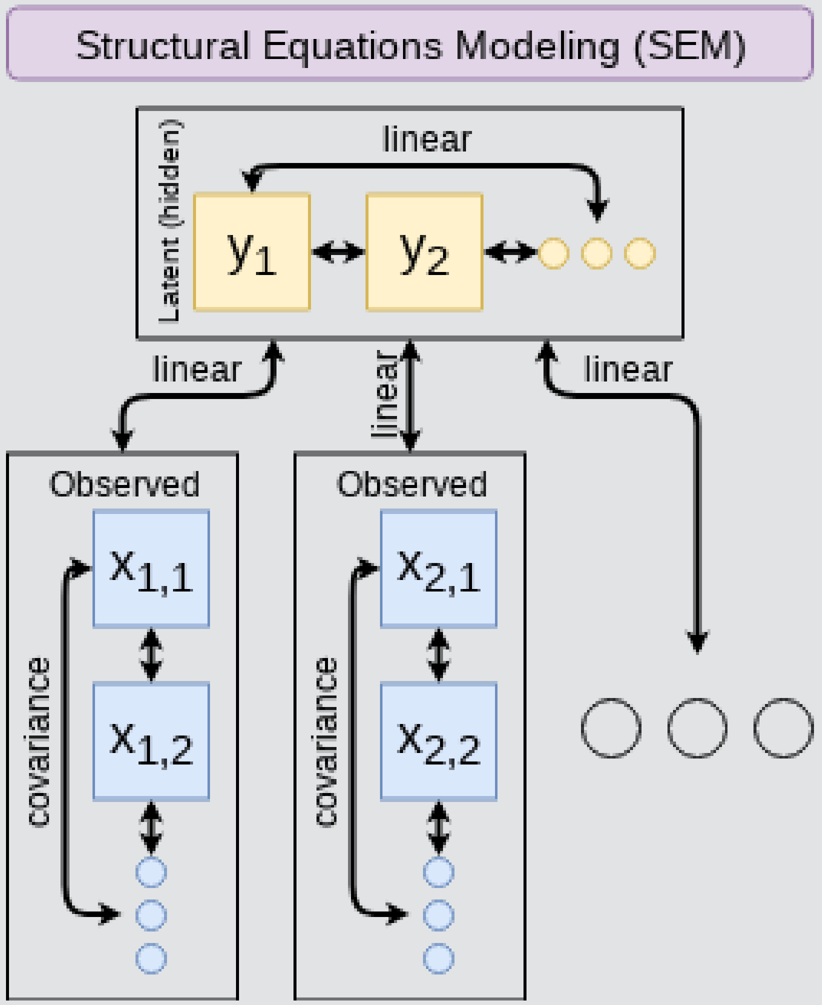
Other linear models: 3SLS



> 3SLS models systems of equations with *related outputs*

- E.g.: Modeling both stock return, volatility, and volume simultaneously

Other linear models: SEM



SEM can model abstract and multi-level relationships

- E.g.: Showing that organizational commitment leads to higher job satisfaction, not the other way around (Poznanski and Bline 1999)

Modeling choices: Model selection

> Pick what fits your problem!

- For forecasting a quantity:
  - Usually some sort of linear model regressed using OLS
  - The other model types mentioned are great for simultaneous forecasting of multiple outputs
- For forecasting a binary outcome:
  - Usually logit or a related model (we'll start this in 2 weeks)
- For forensics:
  - Usually logit or a related model

> There are many more model types though!

Modeling choices: Variable selection

- The options:
  1. Use your own knowledge to select variables
  2. Use a selection model to automate it

Own knowledge

- Build a model based on your knowledge of the problem and situation
- This is generally better
  - The result should be more interpretable
  - For prediction, you should know relationships better than most algorithms

Modeling choices: Automated selection

- Traditional methods include:
  - Forward selection: Start with nothing and add variables with the most contribution to Adj R^2 until it stops going up
  - Backward selection: Start with all inputs and remove variables with the worst (negative) contribution to Adj R^2 until it stops going up
  - Stepwise selection: Like forward selection, but drops non-significant predictors
- Newer methods:
  - Lasso and Elastic Net based models
    - Optimize with high penalties for complexity (i.e., # of inputs)
    - We will discuss these in week 5

The overfitting problem

> Or: Why do we like simpler models so much?

- Overfitting happens when a model fits in-sample data *too well*…
  - To the point where it also models any idiosyncrasies or errors in the data
  - This harms prediction performance
    - Directly harming our forecasts

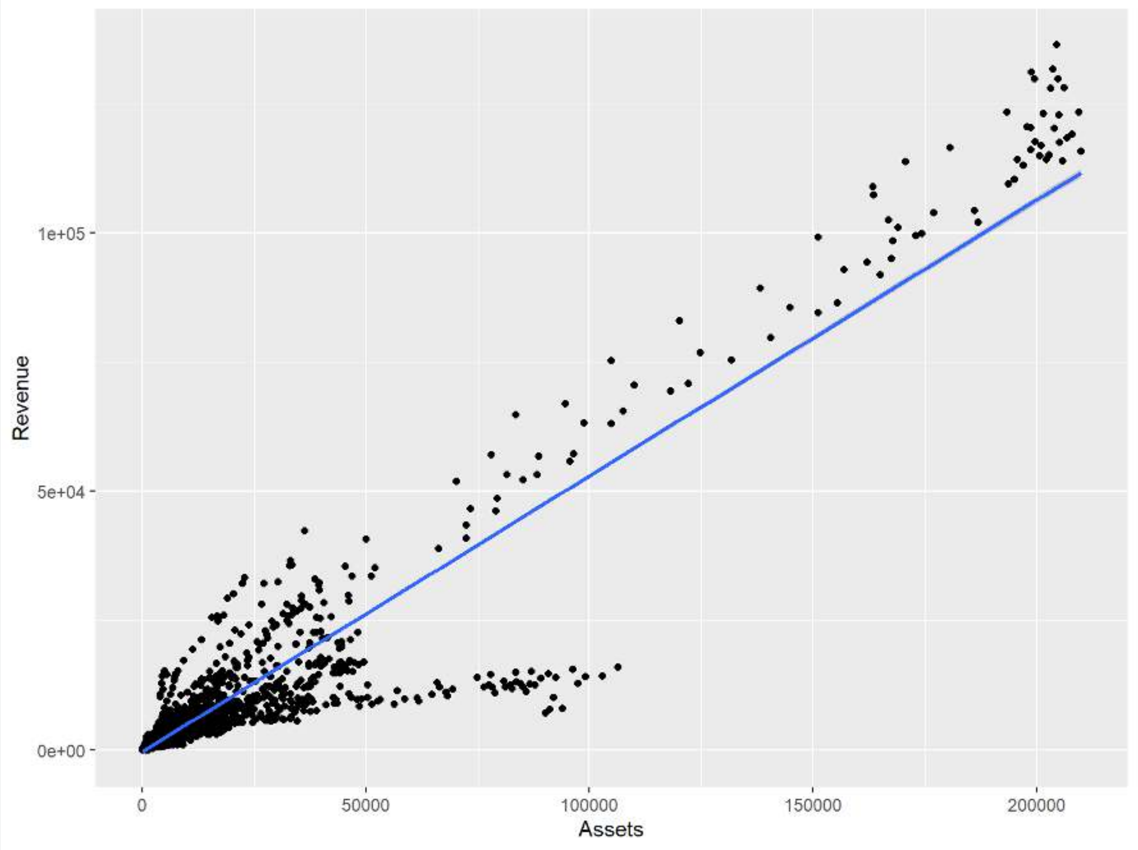
> An overfitted model works really well on its own data, and quite poorly on new data

Statistical tests and Interpretation

Coefficients

- In OLS: β_i

- A change in x_i by 1 leads to a change in y by β_i
- Essentially, the slope between x and y
- The blue line in the chart is the regression line for $\hat{Revenue} = \alpha + \beta_i \hat{Assets}$ for retail firms since 1960



P-values

- p-values tell us the probability that an individual result is due to random chance

> "The P value is defined as the probability under the assumption of no effect or no difference (null hypothesis), of obtaining a result equal to or more extreme than what was actually observed."
> – Dahiru 2008

- These are very useful, particularly for a frequentist approach
- First used in the 1700s, but popularized by Ronald Fisher in the 1920s and 1930s

P-values: Rule of thumb

- If $p < 0.05$ and the coefficient matches our mental model, we can consider this as supporting our model
  - If $p < 0.05$ but the coefficient is opposite, then it is suggesting a problem with our model
  - If $p > 0.10$, it is rejecting the alternative hypothesis
- If $0.05 < p < 0.10$ it depends…
  - For a small dataset or a complex problem, we can use 0.10 as a cutoff
  - For a huge dataset or a simple problem, we should use 0.05

One vs two tailed tests

- Best practice:
  - **Use a two tailed test**
- Second best practice:
  - If you use a 1-tailed test, use a p-value cutoff of 0.025 or 0.05
    - This is equivalent to the best practice, just roundabout
- Common but generally inappropriate:
  - Use a one tailed test with cutoffs of 0.05 or 0.10 because your hypothesis is directional

R^2

- R^2 = Explained variation / Total variation
  - Variation = difference in the observed output variable from its own mean
- A high R^2 indicates that the model fits the data very well
- A low R^2 indicates that the model is missing much of the variation in the output
- R^2 is technically a *biased* estimator
- Adjusted R^2 downweights R^2 and makes it unbiased
  - $R^2_{Adj} = PR^2 + 1 - P$
    - Where $P = \frac{n-1}{n-p-1}$
    - n is the number of observations
    - p is the number of inputs in the model

Test statistics

- Testing a coefficient:
  - Use a t or z test
- Testing a model as a whole
  - F-test, check *adjusted* R squared as well
- Testing across models
  - Chi squared (χ^2) test
  - Vuong test (comparing R^2)
  - Akaike Information Criterion (AIC) (Comparing MLEs, lower is better)

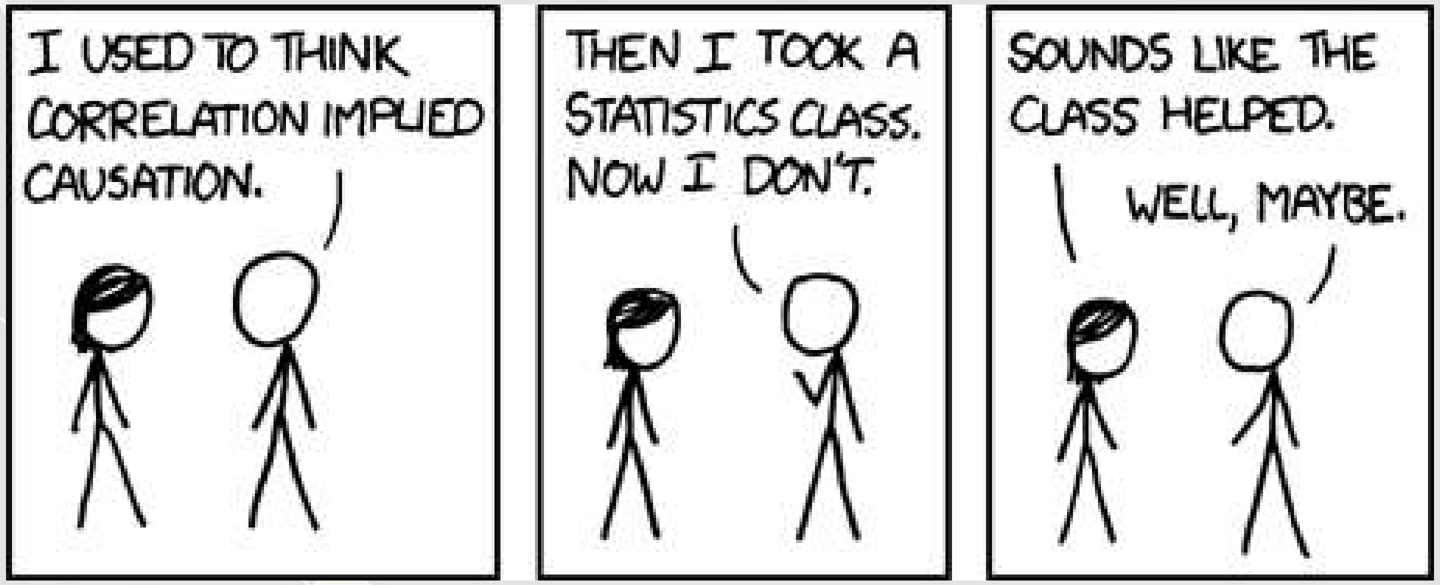
> All of these have p-values, except for AIC

Causality

What is causality?

$$A \to B$$

- Causality is A *causing* B
  - This means more than A and B are correlated
- I.e., If A changes, B changes. But B changing doesn't mean A changed
  - Unless B is 100% driven by A
- Very difficult to determine, particularly for events that happen [almost] simultaneously
- Examples of correlations that aren't causation



Time and causality

$$A \rightarrow B \text{ or } A \leftarrow B?$$

$$A_t \rightarrow B_{t+1}$$

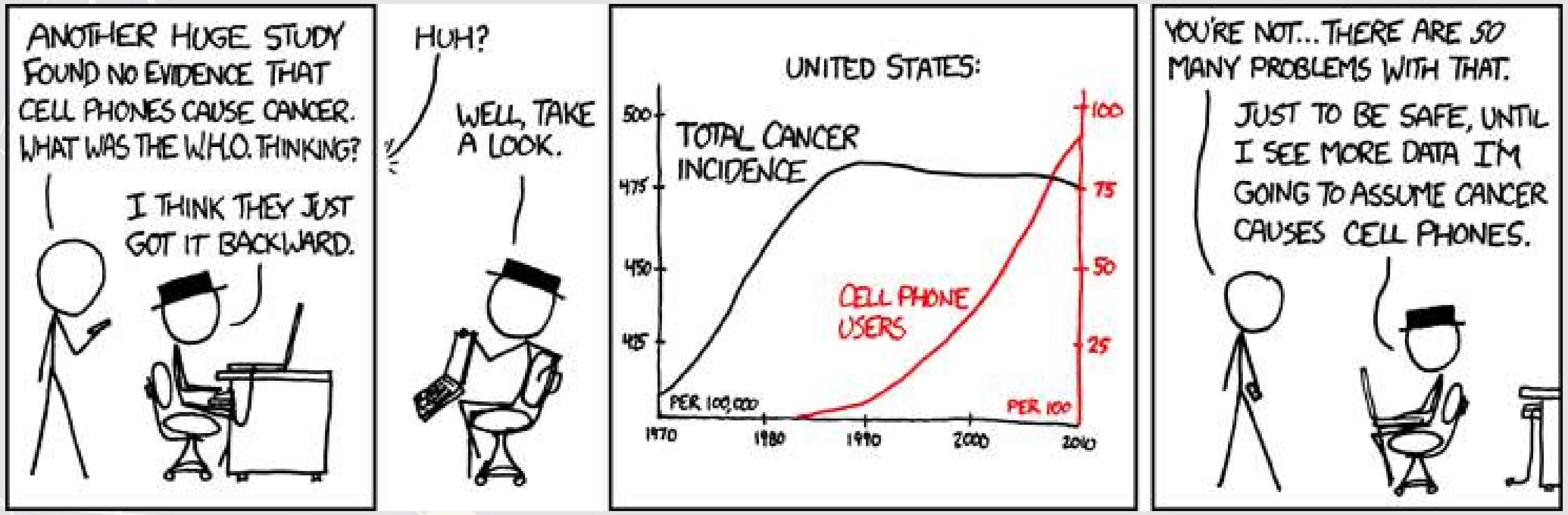
- If there is a separation in time, it's easier to say A caused B
  - Observe A, then see if B changes after
- Conveniently, we have this structure when forecasting
  - Recall last week's model:

$$Revenue_{t+1} = Revenue_t + \ldots$$

Time and causality break down

$$A_t \rightarrow B_{t+1}? \quad \text{OR} \quad C \rightarrow A_t \text{ and } C \rightarrow B_{t+1}?$$

- The above illustrates the *Correlated omitted variable problem*
  - A doesn't cause B… Instead, some other force C causes both
  - Bane of social scientists everywhere
- This is less important for predictive analytics, as we care more about performance, but…
  - It can complicate interpreting your results
  - Figuring out C can help improve you model's predictions
    - So find C!



Revisiting the previous problem

Formalizing our last test

1. Question
   -
2. Hypotheses
   - H_0:
   - H_1:
3. Prediction
   -
4. Testing:
   -
5. Statistical tests:
   - Individual variables:
   - Model:

Is this model better?

```
anova(mod2, mod3, test="Chisq")
```

```
## Analysis of Variance Table
##
## Model 1: revt_growth ~ at_growth
## Model 2: revt_growth ~ lct_growth + che_growth + ebit_growth
##   Res.Df    RSS Df Sum of Sq Pr(>Chi)
## 1     26 1.5534
## 2     24 1.1918  2   0.36168   0.0262 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

> A bit better at $p < 0.05$

- This means our model with change in current liabilities, cash, and EBIT appears to be better than the model with change in assets.

Panel data

Expanding our methodology

- Why should we limit ourselves to 1 firm's data?
- The nature of data analysis is such:

> Adding more data usually helps improve predictions

- Assuming:
  - The data isn't of low quality (too noisy)
  - The data is relevant
  - Any differences can be reasonably controlled for

Expanding our question

- Previously: Can we predict revenue using a firm's accounting information?
  - This is simultaneous, and thus is not forecasting
- Now: Can we predict *future* revenue using a firm's accounting information?
  - By trying to predict ahead, we are now in the realm of forecasting
  - What do we need to change?
    - $\hat{y}$ will need to be 1 year in the future

First things first

- When using a lot of data, it is important to make sure the data is clean
- In our case, we may want to remove any very small firms

```r
# Ensure firms have at least $1M (local currency), and have revenue
# df contains all real estate companies excluding North America
df_clean <- filter(df, df$at>1, df$revt>0)

# We cleaned out 578 observations!
print(c(nrow(df), nrow(df_clean)))
```

```
## [1] 5161 4583
```

```r
# Another useful cleaning funtion:
# Replaces NaN, Inf, and -Inf with NA for all numeric variables in the data!
df_clean <- df_clean %>%
  mutate_if(is.numeric, list(~replace(., !is.finite(.), NA)))
```

Looking back at the prior models

```r
uol <- uol %>% mutate(revt_lead = lead(revt))  # From dplyr
forecast1 <- lm(revt_lead ~ lct + che + ebit, data=uol)
library(broom)  # Lets us view bigger regression outputs in a tidy fashion
tidy(forecast1)  # Present regression output
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)   87.4     124.        0.707  0.486
## 2 lct            0.213     0.291     0.731  0.472
## 3 che            0.112     0.349     0.319  0.752
## 4 ebit           2.49      1.03      2.42   0.0236
```

```r
glance(forecast1)  # Present regression statistics
```

```
## # A tibble: 1 x 11
##   r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>   <dbl> <int>  <dbl> <dbl> <dbl>
## 1     0.655         0.612  357.      15.2 9.39e-6     4  -202.  414.  421.
## # ... with 2 more variables: deviance <dbl>, df.residual <int>
```

> This model is ok, but we can do better.

Expanding the prior model

```r
forecast2 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit , data=uol)
tidy(forecast2)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)  15.6       97.0       0.161 0.874
## 2 revt          1.49       0.414     3.59  0.00174
## 3 act           0.324      0.165     1.96  0.0629
## 4 che           0.0401     0.310     0.129 0.898
## 5 lct          -0.198      0.179    -1.10  0.283
## 6 dp            3.63       5.42      0.669 0.511
## 7 ebit         -3.57       1.36     -2.62  0.0161
```

- Revenue to capture stickiness of revenue
- Current assest & Cash (and equivalents) to capture asset base
- Current liabilities to capture payments due
- Depreciation to capture decrease in real estate asset values
- EBIT to capture operational performance

Expanding the prior model

```r
glance(forecast2)
```

```
## # A tibble: 1 x 11
##   r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>   <dbl> <int>  <dbl> <dbl> <dbl>
## 1     0.903         0.875  203.      32.5 1.41e-9     7  -184.  385.  396.
## # ... with 2 more variables: deviance <dbl>, df.residual <int>
```

```r
anova(forecast1, forecast2, test="Chisq")
```

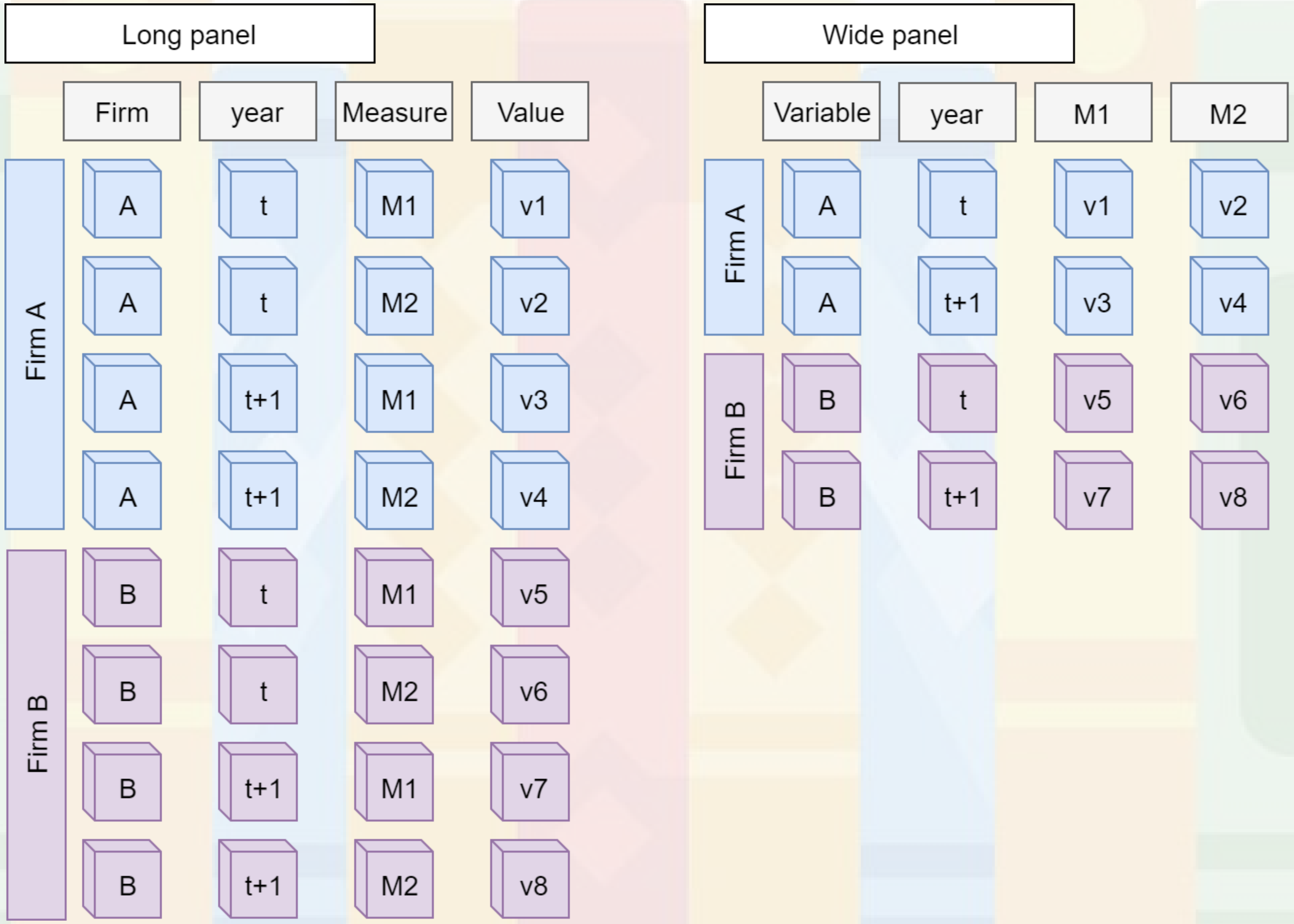
```
## Analysis of Variance Table
##
## Model 1: revt_lead ~ lct + che + ebit
## Model 2: revt_lead ~ revt + act + che + lct + dp + ebit
##   Res.Df     RSS Df Sum of Sq  Pr(>Chi)
## 1     24 3059182
## 2     21  863005  3   2196177 1.477e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This is better (Adj. R^2, χ^2, AIC).

Panel data

- Panel data refers to data with the following characteristics:
  - There is a time dimension
  - There is at least 1 other dimension to the data (firm, country, etc.)
- Special cases:
  - A panel where all dimensions have the same number of observations is called *balanced*
    - Otherwise we call it *unbalanced*
  - A panel missing the time dimension is *cross-sectional*
  - A panel missing the other dimension(s) is a *time series*
- Format:
  - Long: Indexed by all dimensions
  - Wide: Indexed only by other dimensions

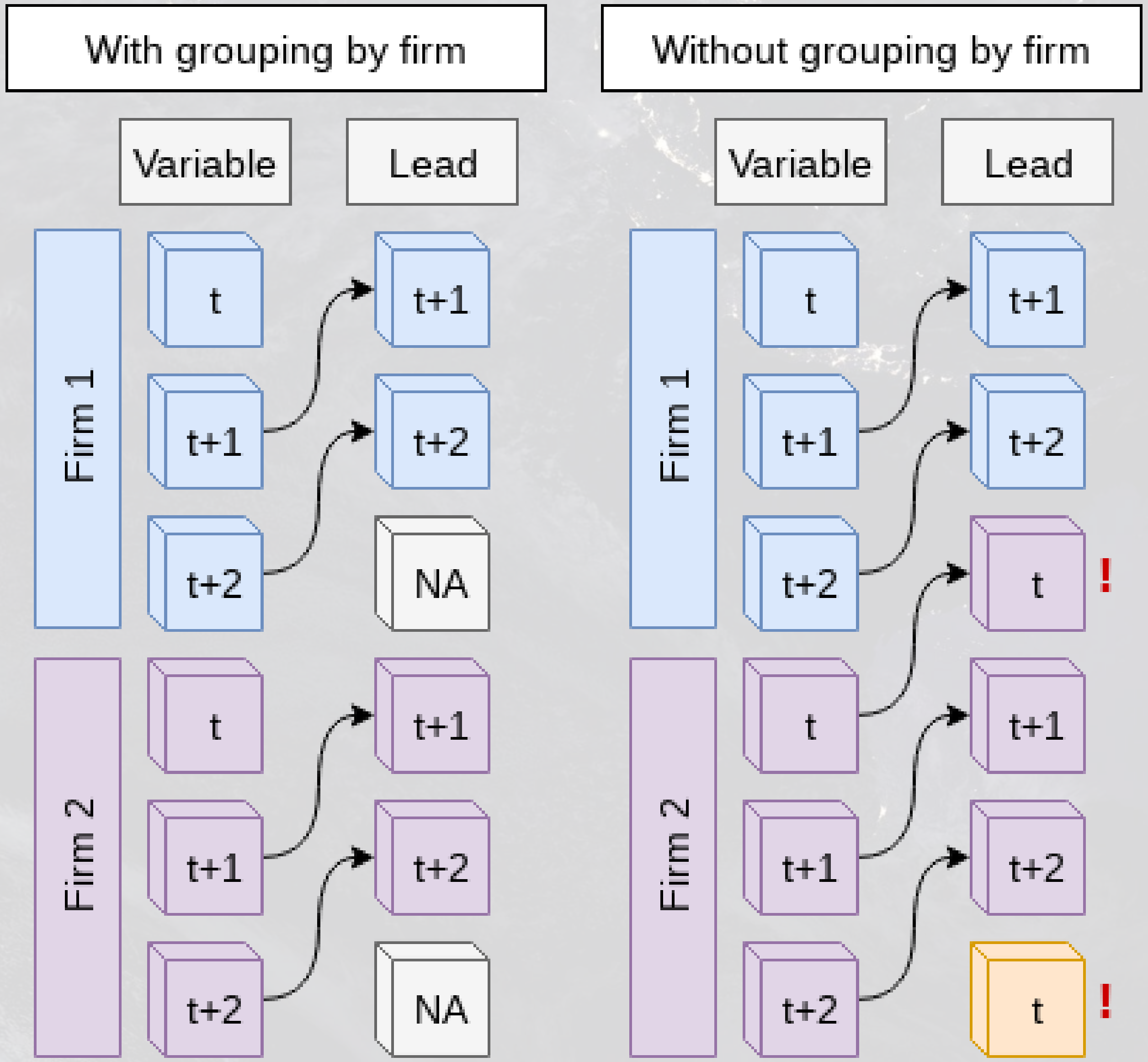
Panel data



Data frames are usually wide panels

All Singapore real estate companies

```r
# Note the group_by -- without it, lead() will pull from the subsequent firm!
# ungroup() tells R that we finished grouping
df_clean <- df_clean %>%
  group_by(isin) %>%
  mutate(revt_lead = lead(revt)) %>%
  ungroup()
```



All Singapore real estate companies

```
forecast3 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit , data=df_clean[df_clean$fic==
tidy(forecast3)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  25.0      13.2         1.89 5.95e- 2
## 2 revt          0.505     0.0762      6.63 1.43e-10
## 3 act          -0.0999    0.0545     -1.83 6.78e- 2
## 4 che           0.494     0.155       3.18 1.62e- 3
## 5 lct           0.396     0.0860      4.60 5.95e- 6
## 6 dp            4.46      1.55        2.88 4.21e- 3
## 7 ebit         -0.951     0.271      -3.51 5.18e- 4
```

All Singapore real estate companies

```r
glance(forecast3)
```

```
## # A tibble: 1 x 11
##   r.squared adj.r.squared sigma statistic   p.value    df logLik   AIC
##       <dbl>         <dbl> <dbl>     <dbl>     <dbl> <int>  <dbl> <dbl>
## 1     0.844         0.841  210.      291. 2.63e-127     7 -2237. 4489.
## # ... with 3 more variables: BIC <dbl>, deviance <dbl>, df.residual <int>
```

> Lower adjusted R^2 – This is worse? Why?

- Note: χ^2 can only be used for models on the same data
  - Same for AIC

Worldwide real estate companies

```
forecast4 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit , data=df_clean)
tidy(forecast4)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) 222.       585.        0.379 7.04e- 1
## 2 revt          0.997     0.00655  152.    0.
## 3 act          -0.00221   0.00547   -0.403 6.87e- 1
## 4 che          -0.150     0.0299    -5.02  5.36e- 7
## 5 lct           0.0412    0.0113     3.64  2.75e- 4
## 6 dp            1.52      0.184      8.26  1.89e-16
## 7 ebit          0.308     0.0650     4.74  2.25e- 6
```

Worldwide real estate companies

```r
glance(forecast4)
```

```
## # A tibble: 1 x 11
##   r.squared adj.r.squared  sigma statistic p.value    df  logLik    AIC
##       <dbl>         <dbl>  <dbl>     <dbl>   <dbl> <int>   <dbl>  <dbl>
## 1     0.944         0.944 36459.    11299.       0     7 -47819. 95654.
## # ... with 3 more variables: BIC <dbl>, deviance <dbl>, df.residual <int>
```

Higher adjusted R^2 – better!

- Note: χ^2 can only be used for models on the same data
  - Same for AIC

Model accuracy

> Why is 1 model better while the other model is worse?

- Ranking:
  1. Worldwide real estate model
  2. UOL model
  3. Singapore real estate model

Dealing with noise

Noise

> Statistical noise is random error in the data

- Many sources of noise:
  - Other factors not included in
  - Error in measurement
    - Accounting measurement!
  - Unexpected events / shocks

> Noise is OK, but the more we remove, the better!

Removing noise: Singapore model

- Different companies may behave slightly differently
  - Control for this using a *Fixed Effect*
  - Note: ISIN uniquely identifies companies

```
forecast3.1 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit + factor(isin),
     data=df_clean[df_clean$fic=="SGP",])
# n=7 to prevent outputting every fixed effect
print(tidy(forecast3.1), n=15)
```

```
## # A tibble: 27 x 5
##    term                       estimate std.error statistic   p.value
##    <chr>                         <dbl>     <dbl>     <dbl>     <dbl>
##  1 (Intercept)                  1.58     39.4       0.0401 0.968
##  2 revt                         0.392     0.0977    4.01   0.0000754
##  3 act                         -0.0538    0.0602   -0.894  0.372
##  4 che                          0.304     0.177     1.72   0.0869
##  5 lct                          0.392     0.0921    4.26   0.0000276
##  6 dp                           4.71      1.73      2.72   0.00687
##  7 ebit                        -0.851     0.327    -2.60   0.00974
##  8 factor(isin)SG1AA6000000 218.        76.5       2.85   0.00463
##  9 factor(isin)SG1AD8000002 -11.7       67.4      -0.174  0.862
## 10 factor(isin)SG1AE2000006   4.02     79.9       0.0503 0.960
## 11 factor(isin)SG1AG0000003 -13.6       61.1      -0.223  0.824
## 12 factor(isin)SG1BG1000000  -0.901    69.5      -0.0130 0.990
## 13 factor(isin)SG1BI9000008   7.76     64.3       0.121  0.904
## 14 factor(isin)SG1DE5000007 -10.8       61.1      -0.177  0.860
## 15 factor(isin)SG1EE1000009  -6.90     66.7      -0.103  0.918
## # ... with 12 more rows
```

Removing noise: Singapore model

```r
glance(forecast3.1)
```

```
## # A tibble: 1 x 11
##   r.squared adj.r.squared sigma statistic   p.value    df logLik   AIC
##       <dbl>         <dbl> <dbl>     <dbl>     <dbl> <int>  <dbl> <dbl>
## 1     0.856         0.844  208.      69.4 1.15e-111    27 -2223. 4502.
## # ... with 3 more variables: BIC <dbl>, deviance <dbl>, df.residual <int>
```

```r
anova(forecast3, forecast3.1, test="Chisq")
```

```
## Analysis of Variance Table
##
## Model 1: revt_lead ~ revt + act + che + lct + dp + ebit
## Model 2: revt_lead ~ revt + act + che + lct + dp + ebit + factor(isin)
##   Res.Df      RSS Df Sum of Sq Pr(>Chi)
## 1    324 14331633
## 2    304 13215145 20   1116488   0.1765
```

> This isn't much different. Why? There is another source of noise within Singapore real estate companies

Another way to do fixed effects

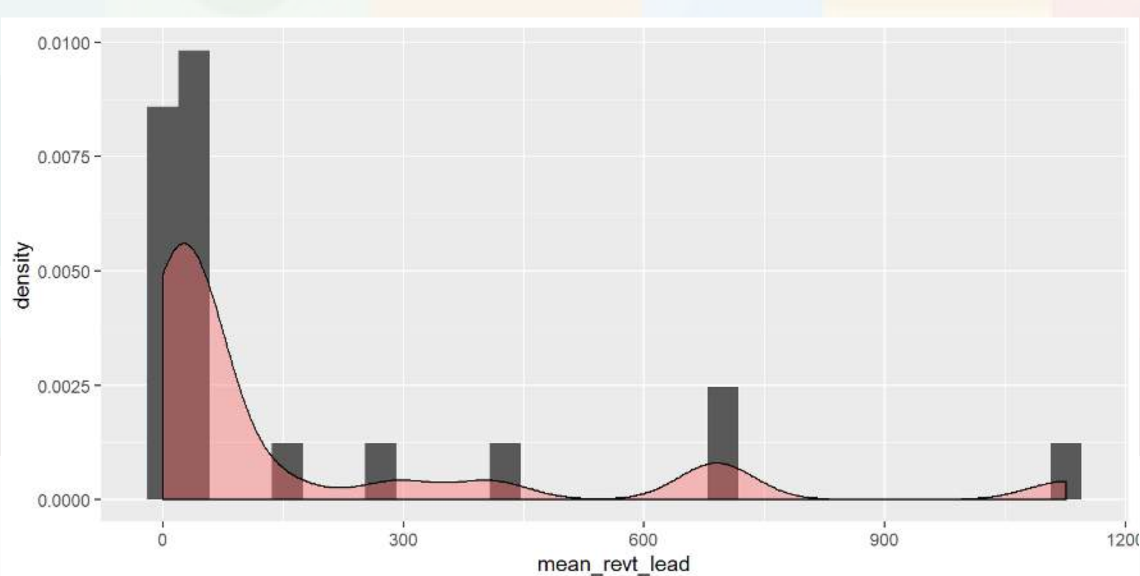
- The library `lfe` has `felm()`: fixed effects linear model
  - Better for complex models
  - Doesn't support prediction natively though

```
library(lfe)
forecast3.2 <-
  felm(revt_lead ~ revt + act + che + lct + dp + ebit | factor(isin),
       data=df_clean[df_clean$fic=="SGP",])
summary(forecast3.2)
```

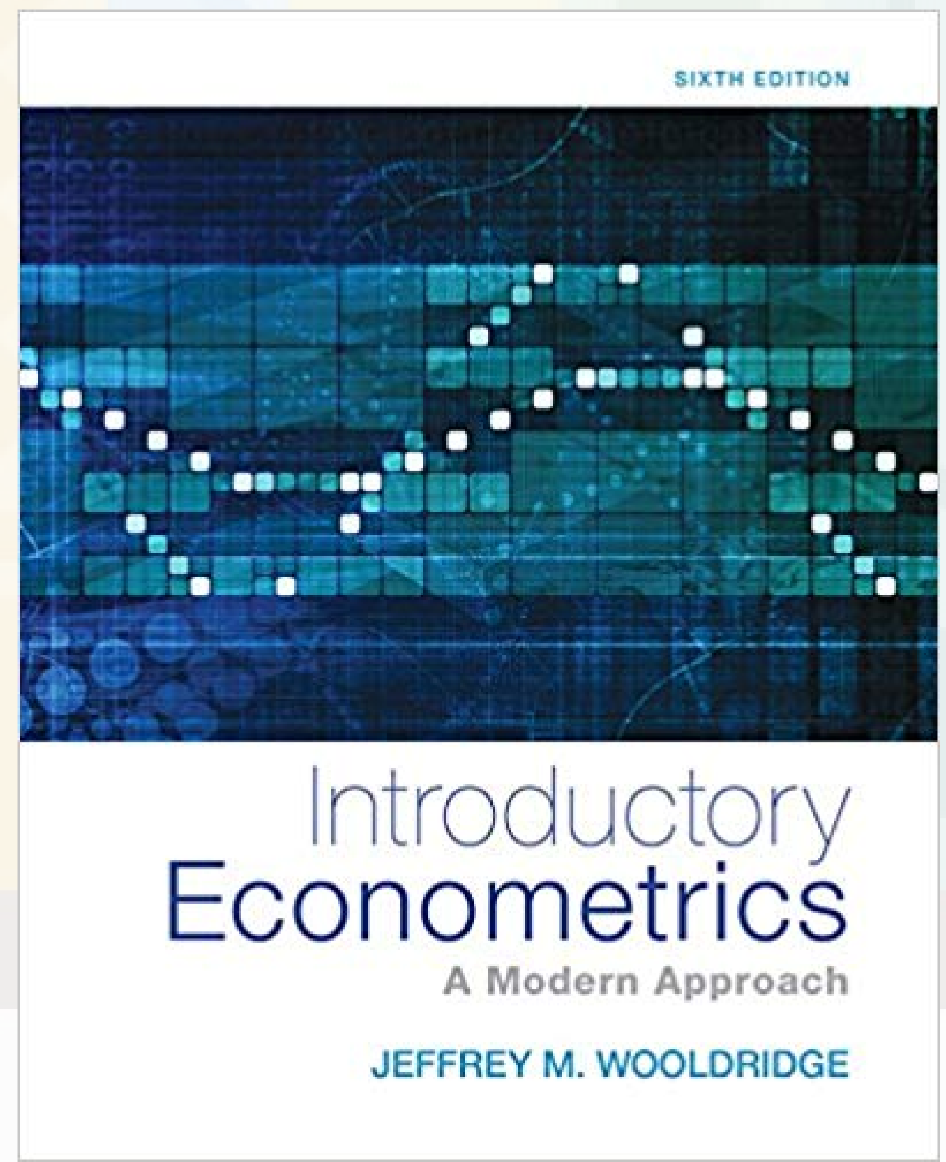
```
##
## Call:
##    felm(formula = revt_lead ~ revt + act + che + lct + dp + ebit |      factor(
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1181.88   -23.25    -1.87    18.03  1968.86
##
## Coefficients:
##      Estimate Std. Error t value Pr(>|t|)
## revt  0.39200    0.09767   4.013 7.54e-05 ***
## act  -0.05382    0.06017  -0.894  0.37181
## che   0.30370    0.17682   1.718  0.08690 .
## lct   0.39209    0.09210   4.257 2.76e-05 ***
## dp    4.71275    1.73168   2.721  0.00687 **
## ebit -0.85080    0.32704  -2.602  0.00974 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

Why exactly would we use fixed effects?

- Fixed effects are used when the average of $\hat{y}$ varies by some group in our data
  - In our problem, the average revenue of each firm is different
- Fixed effects absorb this difference



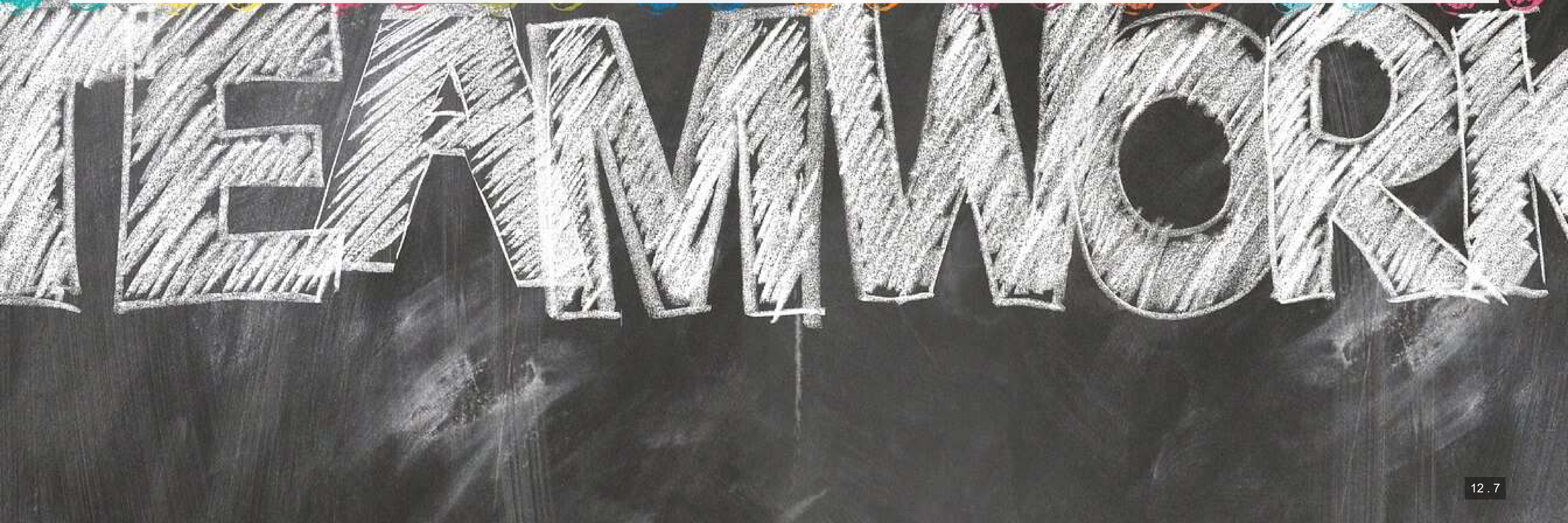
- Further reading:
  - Introductory Econometrics by Jeffrey M. Wooldridge



What else can we do?

> What else could we do to improve our prediction model?

- Assuming:
  1. We do not have access to international data
  2. We do have access to Singaporean firms’ data
  3. We have access to any data that is publicly available



End matter

For next week

- For next week:
  - 2 chapters on Datacamp
  - First individual assignment
    - Do this one individually!
    - Turn in on eLearn before class in 2 weeks

Packages used for these slides

- `broom`
- `DT`
- `knitr`
- `lfe`
- `magrittr`
- `plotly`
- `revealjs`
- `tidyverse`

Custom code

```r
# Graph showing squared error (slide 4.6)
uolg <- uol[,c("at","revt")]
uolg$resid <- mod1$residuals
uolg$xleft <- ifelse(uolg$resid < 0,uolg$at,uolg$at - uolg$resid)
uolg$xright <- ifelse(uolg$resid < 0,uolg$at - uolg$resid, uol$at)
uolg$ytop <- ifelse(uolg$resid < 0,uolg$revt - uolg$resid,uol$revt)
uolg$ybottom <- ifelse(uolg$resid < 0,uolg$revt, uolg$revt - uolg$resid)
uolg$point <- TRUE

uolg2 <- uolg
uolg2$point <- FALSE
uolg2$at <- ifelse(uolg$resid < 0,uolg2$xright,uolg2$xleft)
uolg2$revt <- ifelse(uolg$resid < 0,uolg2$ytop,uolg2$ybottom)

uolg <- rbind(uolg, uolg2)

uolg %>% ggplot(aes(y=revt, x=at, group=point)) +
         geom_point(aes(shape=point)) +
         scale_shape_manual(values=c(NA,18)) +
         geom_smooth(method="lm", se=FALSE) +
         geom_errorbarh(aes(xmax=xright, xmin = xleft)) +
         geom_errorbar(aes(ymax=ytop, ymin = ybottom)) +
         theme(legend.position="none")
```

```r
# Chart of mean revt_lead for Singaporean firms (slide 12.6)
df_clean %>%                                         # Our data frame
  filter(fic=="SGP") %>%                             # Select only Singaporean firms
  group_by(isin) %>%                                 # Group by firm
  mutate(mean_revt_lead=mean(revt_lead, na.rm=T)) %>% # Determine each firm's mean revenue (lead)
  slice(1) %>%                                       # Take only the first observation for each group
  ungroup() %>%                                      # Ungroup (we don't need groups any more)
  ggplot(aes(x=mean_revt_lead)) +                    # Initialize plot and select data
  geom_histogram(aes(y = ..density..)) +             # Plots the histogram as a density so that geom_density is visible
  geom_density(alpha=.4, fill="#FF6666")             # Plots smoothed density
```