ACCT 420: Linear Regression

Session 2

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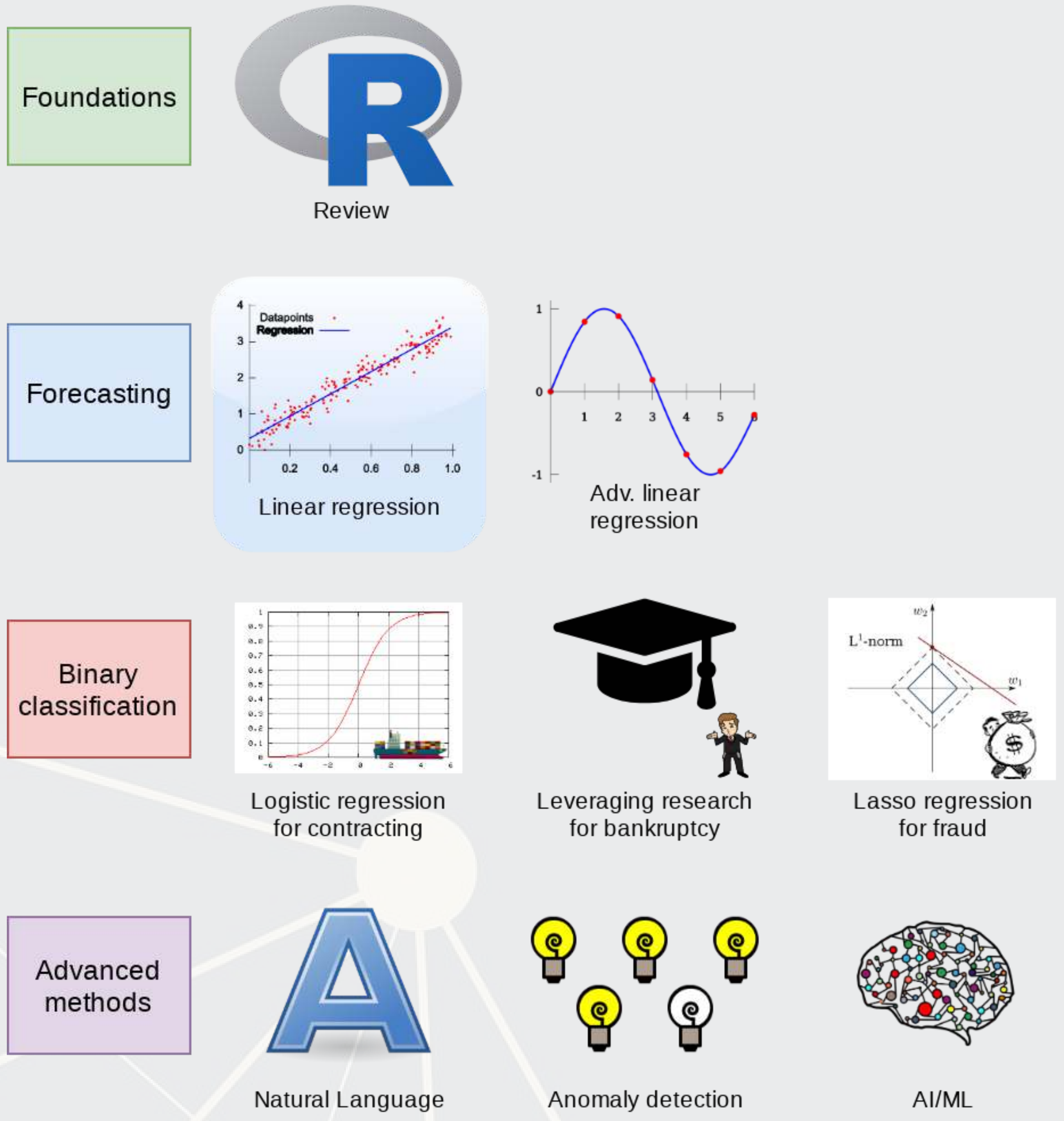
Front matter

Learning objectives



- **Theory:**
  - Develop a logical approach to problem solving with data
    - Statistics
    - Causation
    - Hypothesis testing
- **Application:**
  - Predicting revenue for real estate firms
- **Methodology:**
  - Univariate stats
  - Linear regression
  - Visualization

Datacamp

- For next week:
  - Just 2 chapters:
    - 1 on linear regression
    - 1 on tidyverse methods
- The full list of Datacamp materials for the course is up on eLearn

R Installation

- If you haven't already, make sure to install R and R Studio!
  - Instructions are in Session 1's slides
  - You will need it for this week's assignment
- Please install a few packages using the following code
  - These packages are also needed for the first assignment
  - You are welcome to explore other packages as well, but those will not be necessary for now

```r
# Run this in the R Console inside RStudio
install.packages(c("tidyverse","plotly","tufte"))
```

- Assignments will be provided as R Markdown files

> The format will generally all be filled out – you will just add to it, answer questions, analyze data, and explain your work. Instructions and hints are in the same file

R Markdown: A quick guide

- Headers and subheaders start with `#` and `##`, respectively
- Code blocks starts with `` ```{r} `` and end with `` ``` ``
  - By default, all code and figures will show up in the document
- Inline code goes in a block starting with `` `r `` and ending with `` ` ``
- Italic font can be used by putting `*` or `_` around text
- Bold font can be used by putting `**` around text
  - E.g.: `**bold text**` becomes **bold text**
- To render the document, click Knit
- Math can be placed between `$` to use LaTeX notation
  - E.g. `$\frac{revt}{at}$` becomes $\frac{revt}{at}$
- Full equations (on their own line) can be placed between `$$`
- A block quote is prefixed with `>`
- For a complete guide, see R Studio's R Markdown::Cheat Sheet

Application: Revenue prediction

The question

> How can we predict revenue for a company, leveraging data about that company, related companies, and macro factors

- Specific application: Real estate companies

More specifically…

- Can we use a company’s own accounting data to predict it’s future revenue?
- Can we use other companies’ accounting data to better predict all of their future revenue?
- Can we augment this data with macro economic data to further improve prediction?
  - Singapore business sentiment data

Linear models

What is a linear model?

$$\hat{y} = \alpha + \beta\hat{x} + \varepsilon$$

- The simplest model is trying to predict some outcome $\hat{y}$ as a function of an input $\hat{x}$
  - $\hat{y}$ in our case is a firm's revenue in a given year
  - $\hat{x}$ could be a firm's assets in a given year
  - α and β are solved for
  - ε is the error in the measurement

> I will refer to this as an *OLS* model – **O**rdinary **L**east **S**quare regression

Example

Let's predict UOL's revenue for 2016

- 

- Compustat has data for them since 1989
  - Complete since 1994
    - Missing CapEx before that

```r
# revt: Revenue, at: Assets
summary(uol[,c("revt", "at")])
```

```
##       revt               at       
##  Min.   :  94.78   Min.   : 1218  
##  1st Qu.: 193.41   1st Qu.: 3044  
##  Median : 427.44   Median : 3478  
##  Mean   : 666.38   Mean   : 5534  
##  3rd Qu.:1058.61   3rd Qu.: 7939  
##  Max.   :2103.15   Max.   :19623  
```

Linear models in R

- To run a linear model, use `lm()`
  - The first argument is a formula for your model, where `~` is used in place of an equals sign
    - The left side is what you want to predict
    - The right side is inputs for prediction, separated by `+`
  - The second argument is the data to use
- Additional variations for the formula:
  - Functions transforming inputs (as vectors), such as `log()`
  - Fully interacting variables using `*`
    - I.e., `A*B` includes: `A`, `B`, and `A times B` in the model
  - Interactions using `:`
    - I.e., `A:B` only includes `A times B` in the model

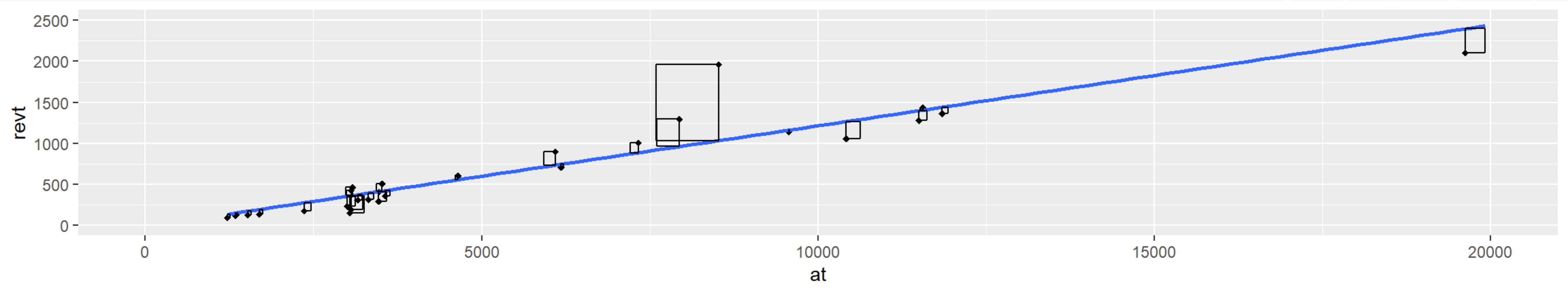
```r
# Example:
lm(revt ~ at, data = uol)
```

Example: UOL

```r
mod1 <- lm(revt ~ at, data = uol)
summary(mod1)
```

```
##
## Call:
## lm(formula = revt ~ at, data = uol)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -295.01 -101.29  -41.09   47.17  926.29
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.831399  67.491305  -0.205    0.839
## at            0.122914   0.009678  12.701  6.7e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 221.2 on 27 degrees of freedom
## Multiple R-squared:  0.8566, Adjusted R-squared:  0.8513
## F-statistic: 161.3 on 1 and 27 DF,  p-value: 6.699e-13
```

Why is it called Ordinary Least *Squares*?



Example: UOL

- This model wasn't so interesting…
  - Bigger firms have more revenue – this is a given
- How about… revenue *growth*?
- And *change* in assets
  - i.e., Asset growth

$$\Delta x_t = \frac{x_t}{x_{t-1}} - 1$$

Calculating changes in R

- The easiest way is using `tidyverse`'s `dplyr`
  - This has a `lag()` function
- The default way to do it is to create a vector manually

```r
# tidyverse
uol <- uol %>%
  mutate(revt_growth1 = revt / lag(revt) - 1)

# R way
uol$revt_growth2 = uol$revt / c(NA, uol$revt[-length(uol$revt)]) - 1

# Check that both ways are equivalent
identical(uol$revt_growth1, uol$revt_growth2)
```

```
## [1] TRUE
```

You can use whichever you are comfortable with

A note on mutate()

- `mutate()` adds variables to an existing data frame
  - If you need to manipulate a bunch of columns at once:
    - `across()` applies a transformation to specified columns in a data frame
    - You can mix in `starts_with()` or `ends_with()` to pick columns by pattern
- Mutate can be very powerful when making more complex variables
  - Examples:
    - Calculating growth within company in a multi-company data frame
    - Normalizing data to be within a certain range for multiple variables at once

Example: UOL with changes

```r
# Make the other needed change
uol <- uol %>%
  mutate(at_growth = at / lag(at) - 1) %>%  # Calculate asset growth
  rename(revt_growth = revt_growth1)        # Rename for readability

# Run the OLS model
mod2 <- lm(revt_growth ~ at_growth, data = uol)
summary(mod2)
```

```
##
## Call:
## lm(formula = revt_growth ~ at_growth, data = uol)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.57736 -0.10534 -0.00953  0.15132  0.42284
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.09024    0.05620   1.606   0.1204
## at_growth    0.53821    0.27717   1.942   0.0631 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2444 on 26 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.1267, Adjusted R-squared:  0.09307
## F-statistic: 3.771 on 1 and 26 DF,  p-value: 0.06307
```

Example: UOL with changes

- ΔAssets doesn't capture ΔRevenue so well
- Perhaps change in total assets is a bad choice?
- Or perhaps we need to expand our model?

Scaling up!

$$\hat{y} = \alpha + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \ldots + \varepsilon$$

- OLS doesn't need to be restricted to just 1 input!
  - Not unlimited though (yet – we'll get there)
    - Number of inputs must be less than the number of observations minus 1
- Each $\hat{x}_i$ is an input in our model
- Each β_i is something we will solve for
- $\hat{y}$, α, and ε are the same as before

Scaling up our model

We have… 464 variables from Compustat Global alone!

- Let’s just add them all?

- We only have 28 observations…
  - 28 << 464…

Now what?

Scaling up our model

> Building a model requires careful thought!

- This is where having accounting and business knowledge comes in!

> What makes sense to add to our model?

Practice: mutate()

- This practice is to make sure you understand how to use mutate with lags
  - These are very important when dealing with business data!
- Do exercises 1 on today's R practice file:
  - R Practice
  - Shortlink: rmc.link/420r2

Statistics Foundations

Frequentist statistics

> A specific test is one of an infinite number of replications

- The “correct” answer should occur most frequently, i.e., with a high probability
- Focus on true vs false
- Treat unknowns as fixed constants to figure out
  - Not random quantities
- Where it’s used
  - Classical statistics methods
    - Like OLS

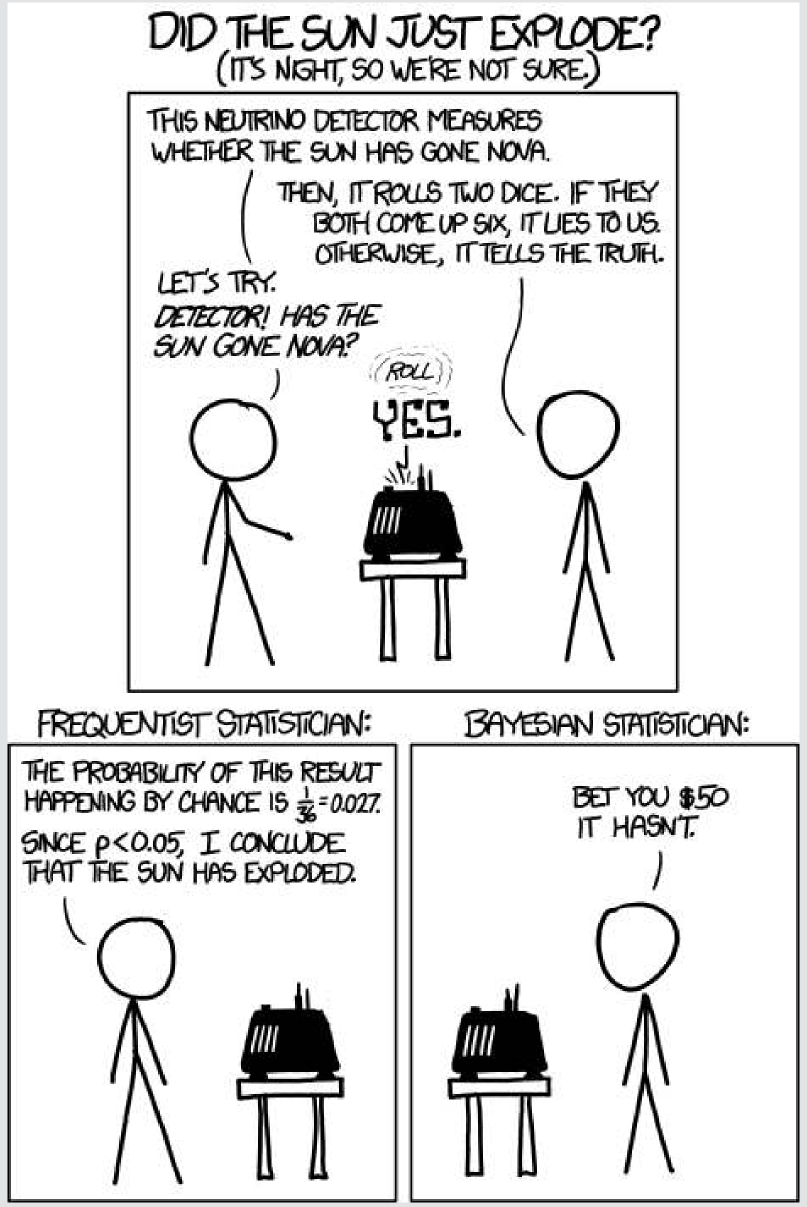
Bayesian statistics

Focus on distributions and beliefs

- Prior distribution – what is believed before the experiment
- Posterior distribution: an updated belief of the distribution due to the experiment
- Derive distributions of parameters
- Where it's used:
  - Many machine learning methods
    - Bayesian updating acts as the learning
  - Bayesian statistics

A separate school of statistics thought

Frequentist vs Bayesian methods



This is why we use more than 1 data point

Frequentist perspective: Repeat the test

```r
detector <- function() {
  dice <- sample(1:6, size=2, replace=TRUE)
  if (sum(dice) == 12) {
    "exploded"
  } else {
    "still there"
  }
}

experiment <- replicate(1000,detector())
# p value
p <- sum(experiment == "still there") / 1000
if (p < 0.05) {
  paste("p-value: ", p, "-- Fail to reject H_A, sun appears to have exploded")
} else {
  paste("p-value: ", p, "-- Reject H_A that sun exploded")
}
```

```
## [1] "p-value:  0.965 -- Reject H_A that sun exploded"
```

Frequentist: The sun didn’t explode

Bayes persepctive: Bayes rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- A: The sun exploded
- B: The detector said it exploded
- $P(A)$: Really, really small. Say, ~0.
- $P(B)$: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
- $P(B|A)$: $\frac{35}{36}$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{\frac{35}{36} \times \sim 0}{\frac{1}{36}} = 35\times \sim 0 \approx 0$$

Bayesian: The sun didn’t explode

What analytics typically relies on

- Regression approaches
  - Most often done in a frequentist manner
  - Can be done in a Bayesian manner as well
- Artificial Intelligence
  - Often frequentist
  - Sometimes neither – “It just works”
- Machine learning
  - Sometimes Bayesian, sometime frequentist
  - We’ll see both

> We will use both to some extent – for our purposes, we will not debate the merits of either school of thought, but we will use tools derived from both

Confusion from frequentist approaches

- Possible contradictions:
  - F test says the model is good yet nothing is statistically significant
  - Individual p-values are good yet the model isn't
  - One measure says the model is good yet another doesn't

> There are many ways to measure a model, each with their own merits. They don't always agree, and it's on us to pick a reasonable measure.

Formalizing frequentist testing

Why formalize?

- Our current approach has been ad hoc
  - What is our goal?
  - How will we know if we have achieved it?
- Formalization provides more rigor

Scientific method

1. Question
   - What are we trying to determine?
2. Hypothesis
   - What do we think will happen? Build a model
3. Prediction
   - What exactly will we test? Formalize model into a statistical approach
4. Testing
   - Test the model
5. Analysis
   - Did it work?

Hypotheses

- Null hypothesis, a.k.a. H_0
  - The status quo
  - Typically: The model *doesn't* work
- Alternative hypothesis, a.k.a. H_1 or H_A
  - The model *does* work (and perhaps how it works)
- Frequentist statistics can never directly support H_0!
  - Only can fail to find support for H_A
  - Even if our p-value is 1, we can't say that the results prove the null hypothesis!

> We will use test statistics to test the hypotheses

Regression

- Regression (like OLS) has the following assumptions
  1. The data is generated following some model
     - E.g., a linear model
       - In two weeks, a logistic model
  2. The data conforms to some statistical properties as required by the test
  3. The model coefficients are something to precisely determine
     - I.e., the coefficients are constants
  4. p-values provide a measure of the chance of an error in a particular aspect of the model
     - For instance, the p-value on β_1 in $y = \alpha + \beta_1 x_1 + \varepsilon$ essentially gives the probability that the sign of β_1 is wrong

OLS Statistical properties

$$\begin{aligned} \text{Theory:} \quad & y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \varepsilon \\ \text{Data:} \quad & \hat{y} = \alpha + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \ldots + \hat{\varepsilon} \end{aligned}$$

1. There should be a *linear* relationship between y and each x_i
   - I.e., y is [approximated by] a constant multiple of each x_i
   - Otherwise we **shouldn't** use a *linear* regression
2. Each $\hat{x}_i$ is normally distributed
   - Not so important with larger data sets, but a good to adhere to
3. Each observation is independent
   - We'll violate this one for the sake of *causality*
4. Homoskedasticity: Variance in errors is constant
   - This is important
5. Not too much multicollinearity
   - Each $\hat{x}_i$ should be relatively independent from the others
   - Some is OK

Practical implications

> Models designed under a frequentist approach can only answer the question of “does this matter?”

- Is this a problem?

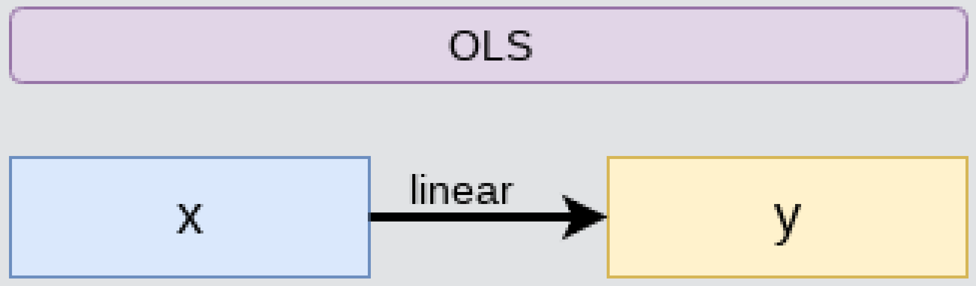
Linear model implementation

What exactly is a linear model?

- Anything OLS is linear
- Many transformations can be recast to linear
  - Ex.: $log(y) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 {x_1}^2 + \beta_4 x_1 \cdot x_2$
    - This is the same as $y' = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ where:
      - $y' = log(y)$
      - $x_3 = {x_1}^2$
      - $x_4 = x_1 \cdot x_2$

> Linear models are *very* flexible

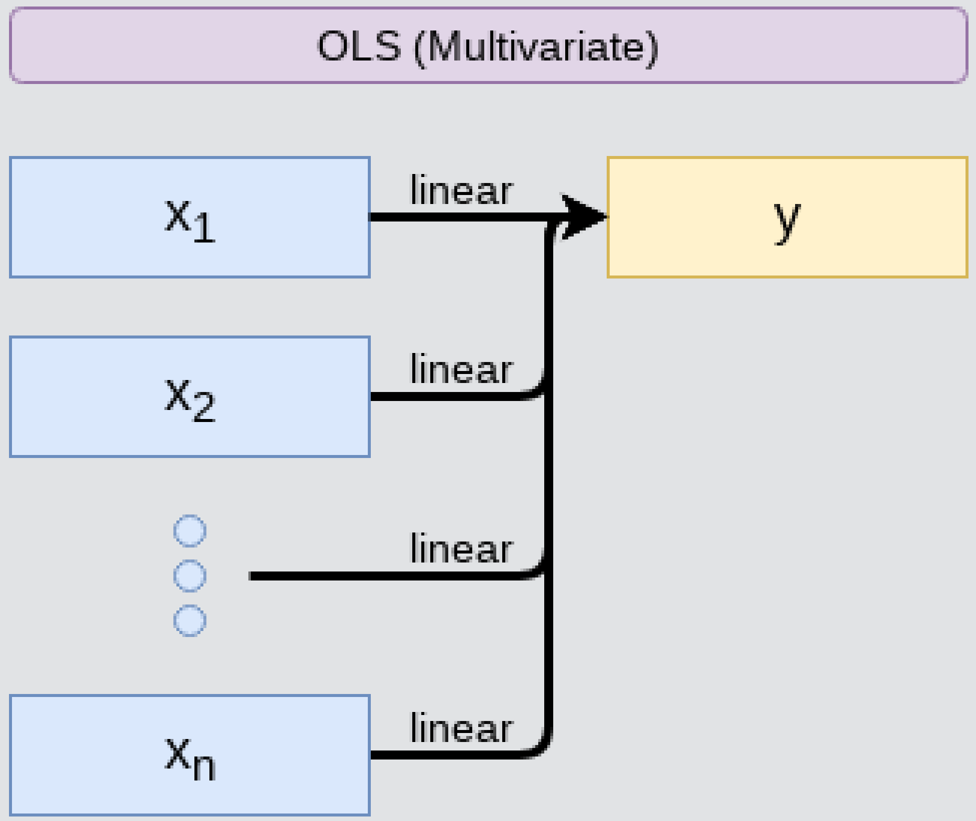
Mental model of OLS: 1 input



> Simple OLS measures a simple linear relationship between an input and an output

- E.g.: Our first regression this week: Revenue on assets

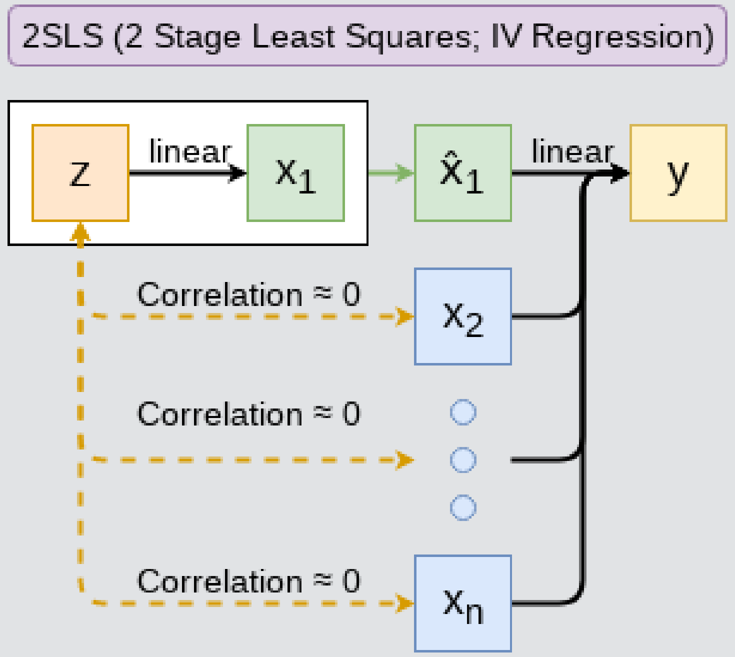
Mental model of OLS: Multiple inputs



> OLS measures simple linear relationships between a set of inputs and one output

- E.g.: This is what we did when scaling up earlier this session

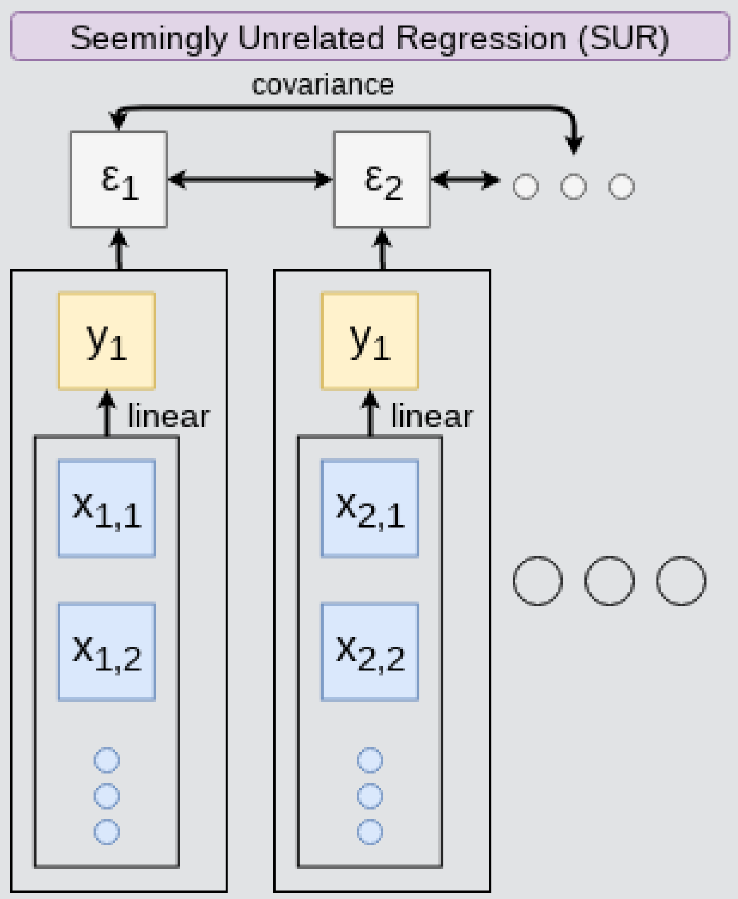
Other linear models: IV Regression (2SLS)



> IV/2SLS models linear relationships where the effect of some x_i on y may be confounded by outside factors.

- E.g.: Modeling the effect of management pay duration (like bond duration) on firms' choice to issue earnings forecasts
  - Instrument with CEO tenure (Cheng, Cho, and Kim 2015)

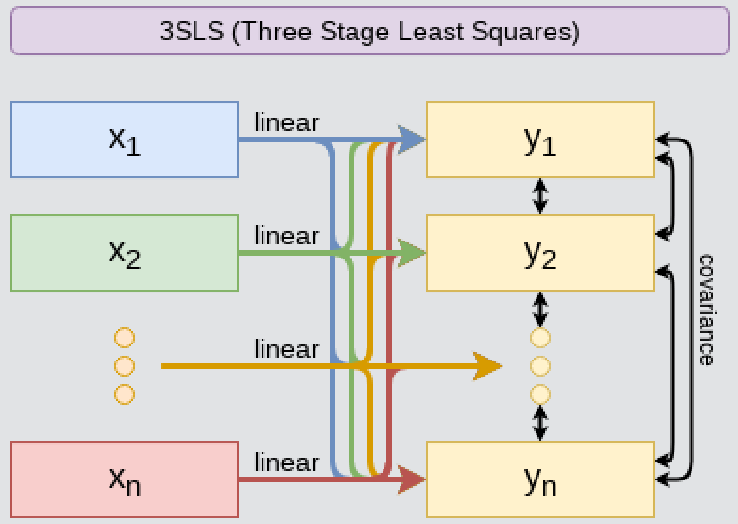
Other linear models: SUR



SUR models systems with related error terms

- E.g.: Modeling both revenue and earnings simultaneously

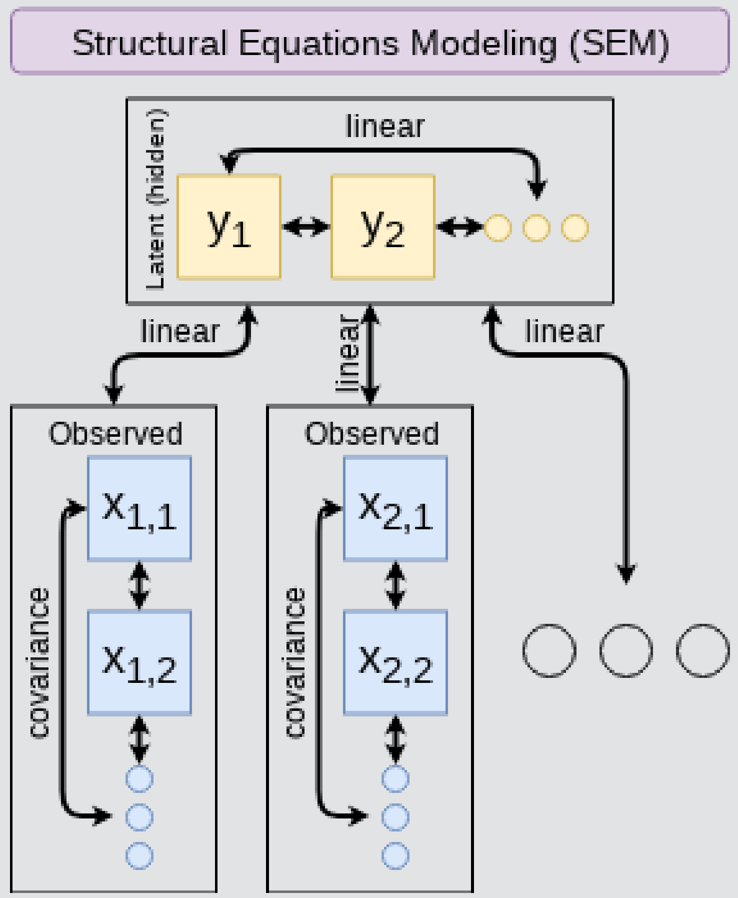
Other linear models: 3SLS



> 3SLS models systems of equations with *related outputs*

- E.g.: Modeling stock return, volatility, and volume simultaneously

Other linear models: SEM



> SEM can model abstract and multi-level relationships

- E.g.: Showing that organizational commitment leads to higher job satisfaction, not the other way around (Poznanski and Bline 1999)

Modeling choices: Model selection

Pick what fits your problem!

- For forecasting a quantity:
  - Usually some sort of linear model regressed using OLS
  - The other model types mentioned are great for simultaneous forecasting of multiple outputs
- For forecasting a binary outcome:
  - Usually logit or a related model (we'll start this in 2 weeks)
- For forensics:
  - Usually logit or a related model

There are many more model types though!

Modeling choices: Variable selection

- The options:
  1. Use your own knowledge to select variables
  2. Use a selection model to automate it

Own knowledge

- Build a model based on your knowledge of the problem and situation
- This is generally better
  - The result should be more interpretable
  - For prediction, you should know relationships better than most algorithms

Modeling choices: Automated selection

- Traditional methods include:
  - Forward selection: Start with nothing and add variables with the most contribution to Adj R^2 until it stops going up
  - Backward selection: Start with all inputs and remove variables with the worst (negative) contribution to Adj R^2 until it stops going up
  - Stepwise selection: Like forward selection, but drops non-significant predictors
- Newer methods:
  - Lasso and Elastic Net based models
    - Optimize with high penalties for complexity (i.e., # of inputs)
    - We will discuss these in week 5
    - These are proven to be better

The overfitting problem

Or: Why do we like simpler models so much?

- Overfitting happens when a model fits in-sample data *too well*…
  - To the point where it also models any idiosyncrasies or errors in the data
  - This harms prediction performance
    - Directly harming our forecasts

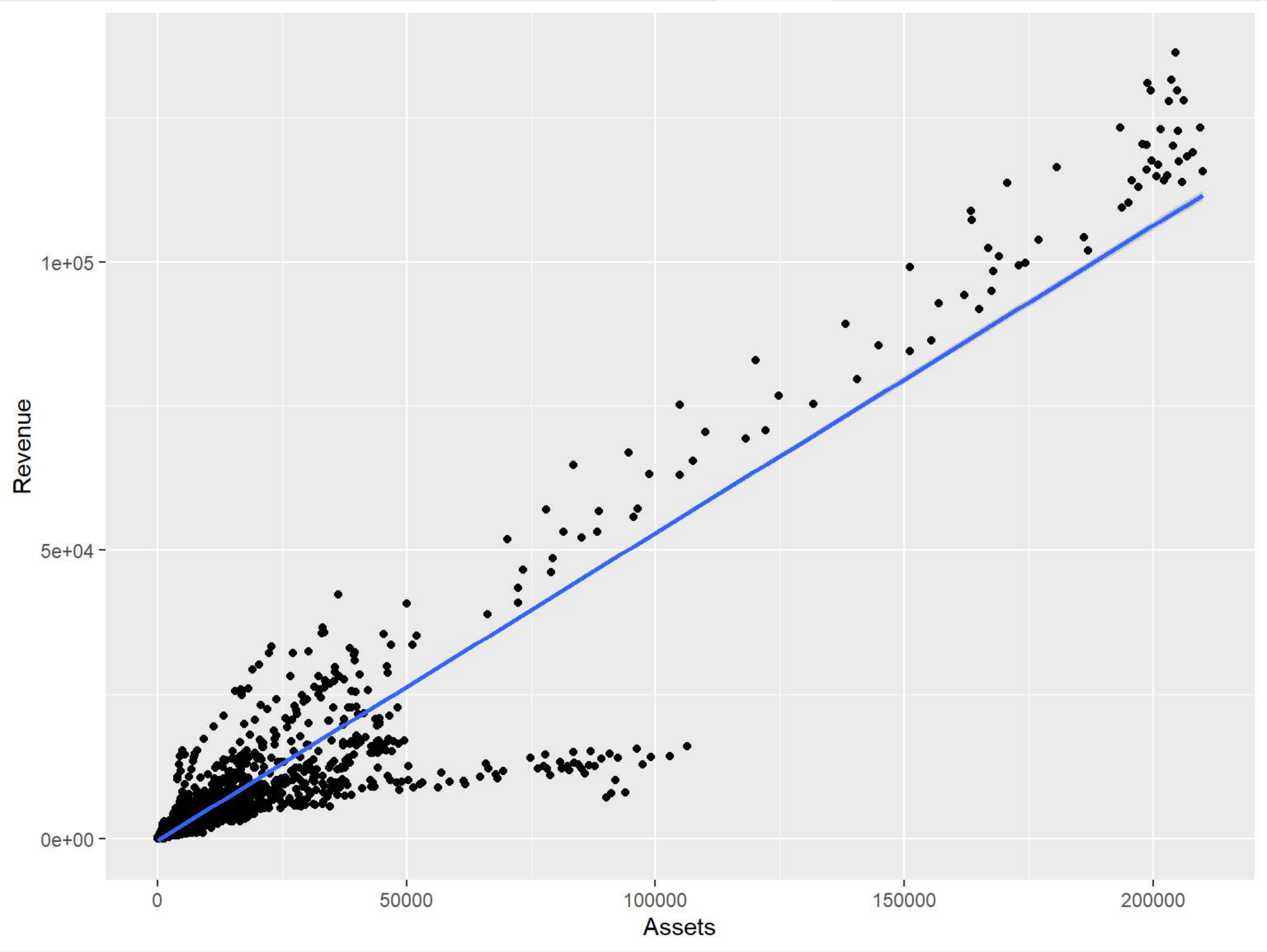
An overfitted model works really well on its own data, and quite poorly on new data

Statistical tests and Interpretation

Coefficients

- In OLS: β_i

- A change in x_i by 1 leads to a change in y by β_i
- Essentially, the slope between x and y
- The blue line in the chart is the regression line for $\hat{Revenue} = \alpha + \beta_i \hat{Assets}$ for retail firms since 1960



P-values

- p-values tell us the probability that an individual result is due to random chance

> "The P value is defined as the probability under the assumption of no effect or no difference (null hypothesis), of obtaining a result equal to or more extreme than what was actually observed."
> – Dahiru 2008

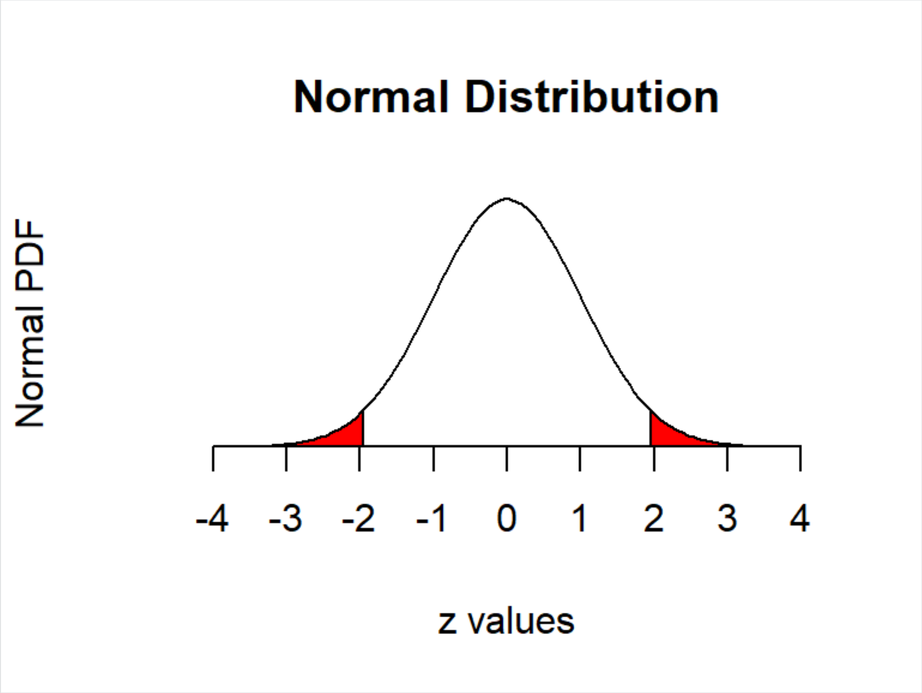
- These are very useful, particularly for a frequentist approach
- First used in the 1700s, but popularized by Ronald Fisher in the 1920s and 1930s

P-values: Rule of thumb

- If $p < 0.05$ and the coefficient sign matches our mental model, we can consider this as supporting our model
  - If $p < 0.05$ but the coefficient is opposite, then it is suggesting a problem with our model
  - If $p > 0.10$, it is rejecting the alternative hypothesis
- If $0.05 < p < 0.10$ it depends…
  - For a small dataset or a complex problem, we can use 0.10 as a cutoff
  - For a huge dataset or a simple problem, we should use 0.05
    - We may even set a lower threshold if we have a ton of data

One vs two tailed tests

- Best practice:
  - **Use a two tailed test**
- Second best practice:
  - If you use a 1-tailed test, use a p-value cutoff of 0.025 or 0.05
    - This is equivalent to the best practice, just roundabout
- Common but generally inappropriate:
  - Use a one tailed test with cutoffs of 0.05 or 0.10 because your hypothesis is directional



R^2

- R^2 = Explained variation / Total variation
  - Variation = difference in the observed output variable from its own mean
- A high R^2 indicates that the model fits the data very well
- A low R^2 indicates that the model is missing much of the variation in the output
- R^2 is technically a *biased* estimator
- Adjusted R^2 downweights R^2 and makes it unbiased
  - $R^2_{Adj} = PR^2 + 1 - P$
    - Where $P = \frac{n-1}{n-p-1}$
    - n is the number of observations
    - p is the number of inputs in the model

Test statistics

- Testing a coefficient:
  - Use a t or z test
- Testing a model as a whole
  - F-test, check *adjusted* R squared as well
- Testing across models
  - Chi squared (χ^2) test
  - Vuong test (comparing R^2)
  - Akaike Information Criterion (AIC) (Comparing MLEs, lower is better)

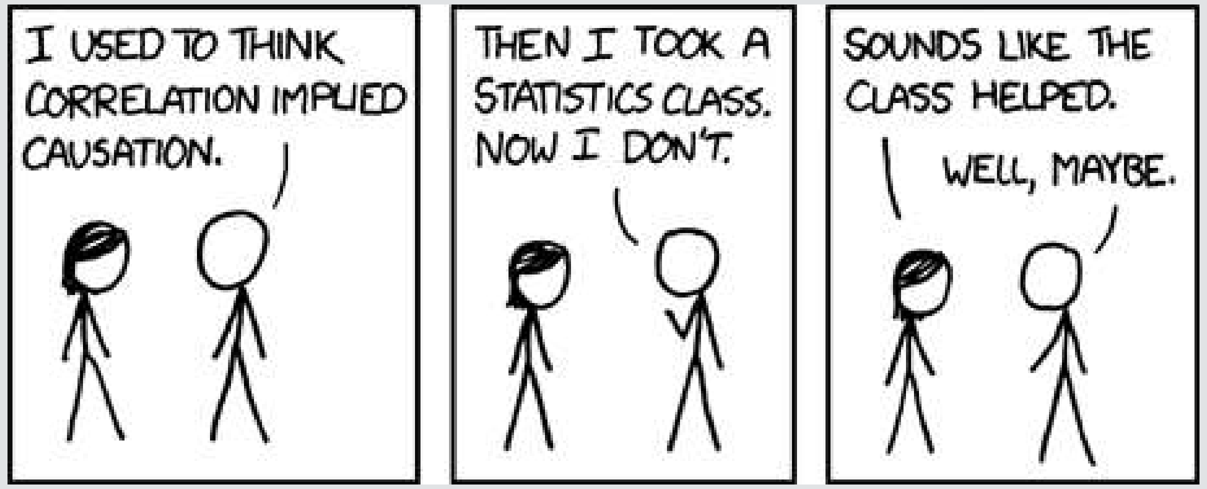
> All of these have p-values, except for AIC

Causality

What is causality?

$$A \to B$$

- Causality is A *causing* B
  - This means more than A and B are correlated
- I.e., If A changes, B changes. But B changing doesn't mean A changed
  - Unless B is 100% driven by A
- Very difficult to determine, particularly for events that happen [almost] simultaneously
- Examples of correlations that aren't causation



Time and causality

$A \rightarrow B$ or $A \leftarrow B$?

$$A_t \rightarrow B_{t+1}$$

- If there is a separation in time, it's easier to say A caused B
  - Observe A, then see if B changes after
- Conveniently, we have this structure when forecasting
  - Consider a model like:

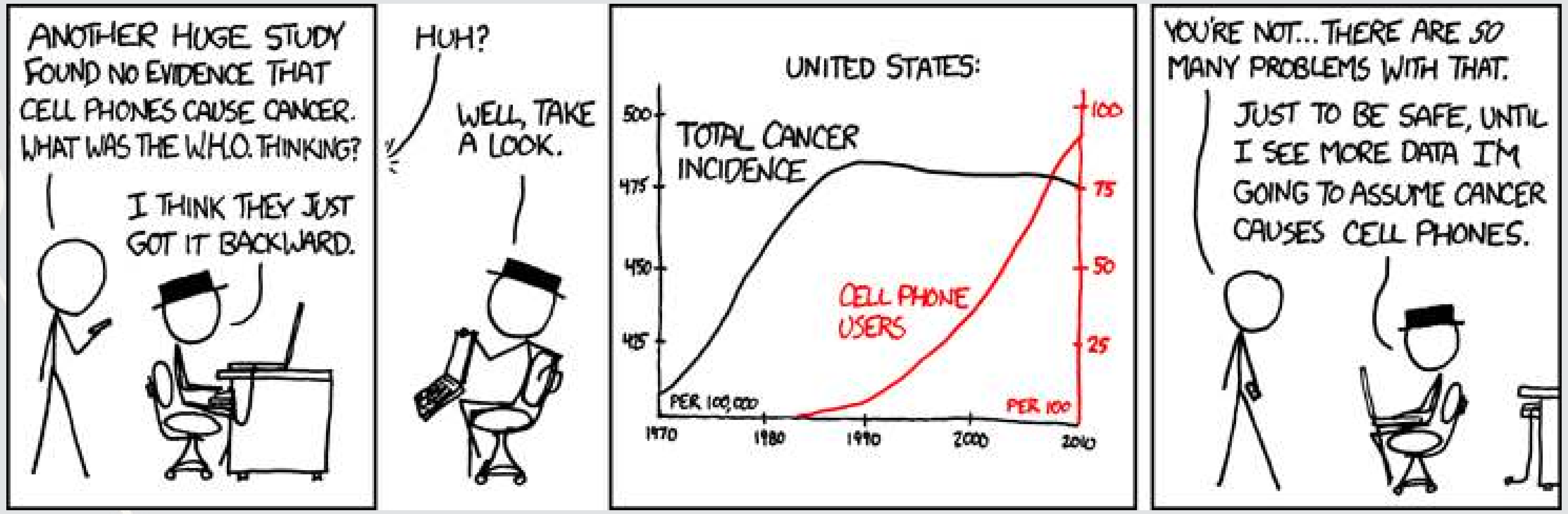
$$Revenue_{t+1} = Revenue_t + \ldots$$

> It would be quite difficult for $Revenue_{t+1}$ to cause $Revenue_t$

Time and causality break down

$$A_t \rightarrow B_{t+1}? \quad \text{OR} \quad C \rightarrow A_t \text{ and } C \rightarrow B_{t+1}?$$

- The above illustrates the *Correlated omitted variable problem*
  - A doesn't cause B… Instead, some other force C causes both
  - The bane of social scientists everywhere
- This is less important for predictive analytics, as we care more about performance, but…
  - It can complicate interpreting your results
  - Figuring out C can help improve you model's predictions
    - So find C!



Revisiting the previous problem

Formalizing our last test

1. Question
   -
2. Hypotheses
   - H_0:
   - H_1:
3. Prediction
   -
4. Testing:
   -
5. Statistical tests:
   - Individual variables:
   - Model:

Is this model better?

```r
anova(mod2, mod3, test="Chisq")
```

```
## Analysis of Variance Table
##
## Model 1: revt_growth ~ at_growth
## Model 2: revt_growth ~ lct_growth + che_growth + ebit_growth
##   Res.Df    RSS Df Sum of Sq Pr(>Chi)
## 1     26 1.5534
## 2     24 1.1918  2   0.36168   0.0262 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

A bit better at $p < 0.05$

- This means our model with change in current liabilities, cash, and EBIT appears to be better than the model with change in assets.

Panel data

Expanding our methodology

- Why should we limit ourselves to 1 firm's data?
- The nature of data analysis is such:

> Adding more data usually helps improve predictions

- Assuming:
  - The data isn't of low quality (too noisy)
  - The data is relevant
  - Any differences can be reasonably controlled for

Expanding our question

- Previously: Can we predict revenue using a firm’s accounting information?
  - This is simultaneous, and thus is not forecasting
- Now: Can we predict *future* revenue using a firm’s accounting information?
  - By trying to predict ahead, we are now in the realm of forecasting
  - What do we need to change?
    - $\hat{y}$ will need to be 1 year in the future

First things first

- When using a lot of data, it is important to make sure the data is clean
- In our case, we may want to remove any very small firms

```r
# Ensure firms have at least $1M (local currency), and have revenue
# df contains all real estate companies excluding North America
df_clean <- df %>%
  filter(at>1, revt>0)

# We cleaned out 578 observations!
print(c(nrow(df), nrow(df_clean)))
```

```
## [1] 5161 4583
```

```r
# Another useful cleaning funtion:
# Replaces NaN, Inf, and -Inf with NA for all numeric variables in the data!
df_clean <- df_clean %>%
  mutate_if(is.numeric, list(~replace(., !is.finite(.), NA)))
```

Looking back at the prior models

```r
uol <- uol %>% mutate(revt_lead = lead(revt))  # From dplyr
forecast1 <- lm(revt_lead ~ lct + che + ebit, data=uol)
library(broom)  # Lets us view bigger regression outputs in a tidy fashion
tidy(forecast1)  # Present regression output
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)   87.4      124.       0.707  0.486
## 2 lct            0.213     0.291     0.731  0.472
## 3 che            0.112     0.349     0.319  0.752
## 4 ebit           2.49      1.03      2.42   0.0236
```

```r
glance(forecast1)  # Present regression statistics
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic    p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>      <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.655         0.612  357.      15.2 0.00000939     3  -202.  414.  421.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

This model is ok, but we can do better.

Expanding the prior model

```
forecast2 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit , data=uol)
tidy(forecast2)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)  15.6       97.0       0.161 0.874
## 2 revt          1.49       0.414     3.59  0.00174
## 3 act           0.324      0.165     1.96  0.0629
## 4 che           0.0401     0.310     0.129 0.898
## 5 lct          -0.198      0.179    -1.10  0.283
## 6 dp            3.63       5.42      0.669 0.511
## 7 ebit         -3.57       1.36     -2.62  0.0161
```

- Revenue to capture stickiness of revenue
- Current assest & Cash (and equivalents) to capture asset base
- Current liabilities to capture payments due
- Depreciation to capture decrease in real estate asset values
- EBIT to capture operational performance

Expanding the prior model

```r
glance(forecast2)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic       p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>         <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.903         0.875  203.      32.5 0.00000000141     6  -184.  385.  396.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
anova(forecast1, forecast2, test="Chisq")
```

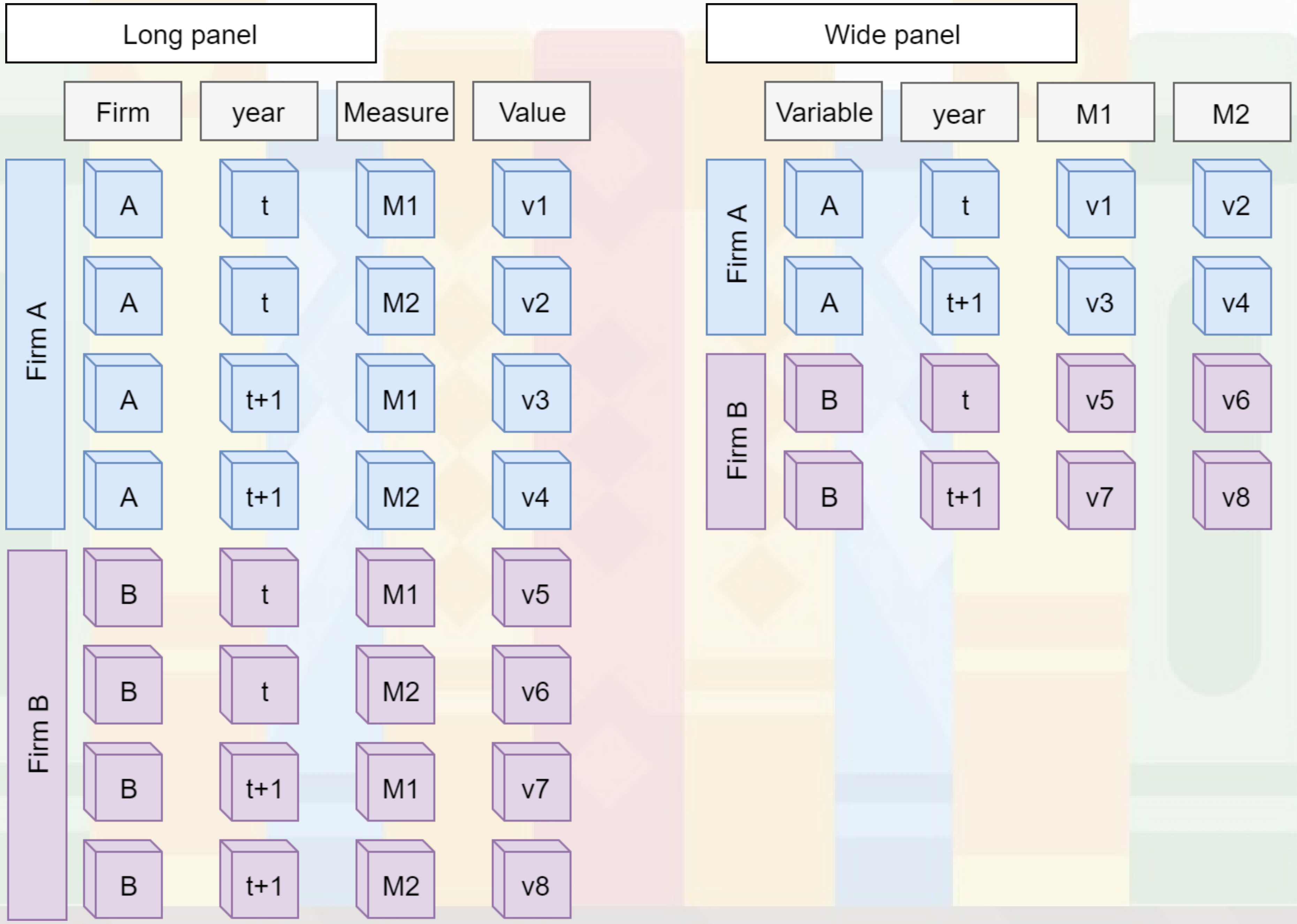
```
## Analysis of Variance Table
##
## Model 1: revt_lead ~ lct + che + ebit
## Model 2: revt_lead ~ revt + act + che + lct + dp + ebit
##   Res.Df     RSS Df Sum of Sq  Pr(>Chi)
## 1     24 3059182
## 2     21  863005  3   2196177 1.477e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This is better (Adj. R^2, χ^2, AIC).

Panel data

- Panel data refers to data with the following characteristics:
  - There is a time dimension
  - There is at least 1 other dimension to the data (firm, country, etc.)
- Special cases:
  - A panel where all dimensions have the same number of observations is called *balanced*
    - Otherwise we call it *unbalanced*
  - A panel missing the time dimension is *cross-sectional*
  - A panel missing the other dimension(s) is a *time series*
- Format:
  - Long: Indexed by all dimensions
  - Wide: Indexed only by some dimensions

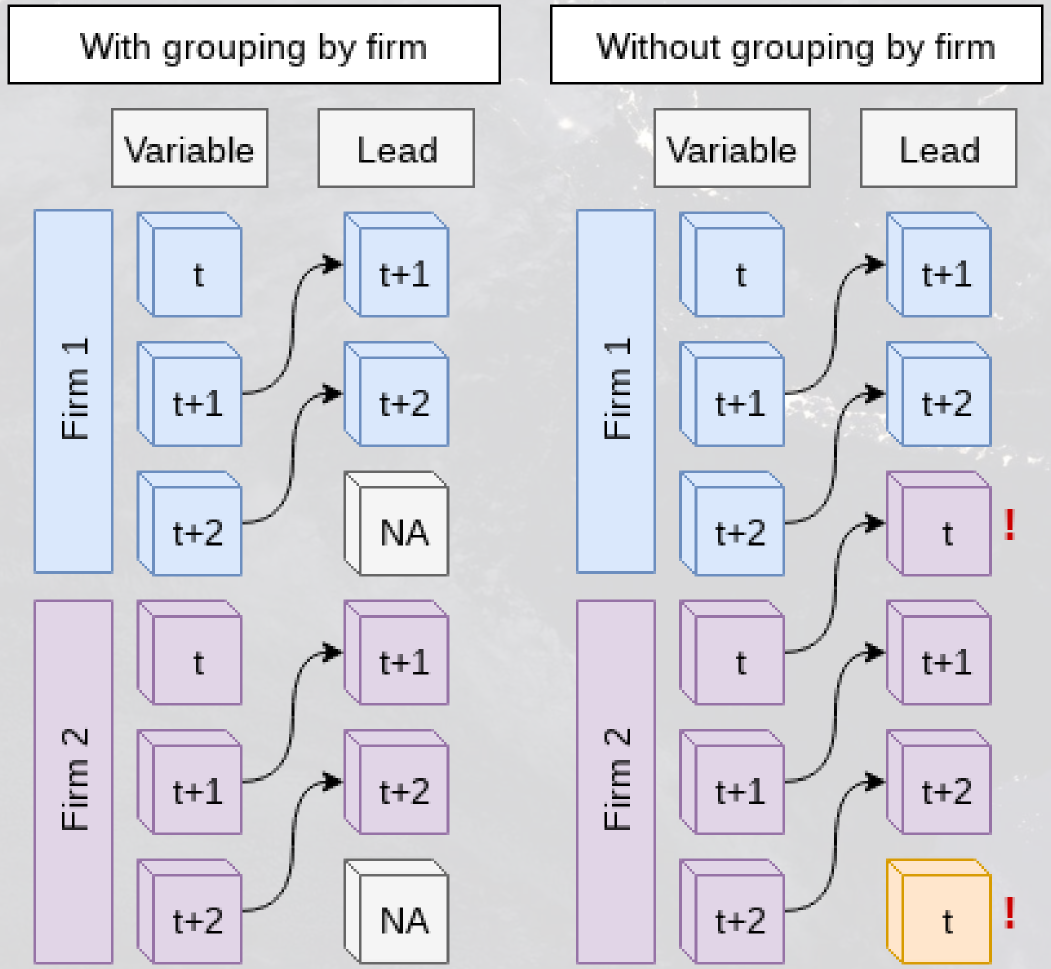
Panel data



Data frames are usually wide panels

All Singapore real estate companies

```r
# Note the group_by -- without it, lead() will pull from the subsequent firm!
# ungroup() tells R that we finished grouping
df_clean <- df_clean %>%
  group_by(isin) %>%
  mutate(revt_lead = lead(revt)) %>%
  ungroup()
```



All Singapore real estate companies

```r
forecast3 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit,
     data=df_clean[df_clean$fic=="SGP",])
tidy(forecast3)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  25.0      13.2         1.89 5.95e- 2
## 2 revt          0.505     0.0762      6.63 1.43e-10
## 3 act          -0.0999    0.0545     -1.83 6.78e- 2
## 4 che           0.494     0.155       3.18 1.62e- 3
## 5 lct           0.396     0.0860      4.60 5.95e- 6
## 6 dp            4.46      1.55        2.88 4.21e- 3
## 7 ebit         -0.951     0.271      -3.51 5.18e- 4
```

All Singapore real estate companies

```
glance(forecast3)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic   p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>     <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.844         0.841  210.      291. 2.63e-127     6 -2237. 4489. 4519.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Lower adjusted R^2 – This is worse? Why?

- Note: χ^2 can only be used for models on the same data
  - Same for AIC

Worldwide real estate companies

```r
forecast4 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit , data=df_clean)
tidy(forecast4)
```

```
## # A tibble: 7 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) 222.      585.          0.379 7.04e- 1
## 2 revt          0.997     0.00655   152.    0
## 3 act          -0.00221   0.00547    -0.403 6.87e- 1
## 4 che          -0.150     0.0299     -5.02  5.36e- 7
## 5 lct           0.0412    0.0113      3.64  2.75e- 4
## 6 dp            1.52      0.184       8.26  1.89e-16
## 7 ebit          0.308     0.0650      4.74  2.25e- 6
```

Worldwide real estate companies

```r
glance(forecast4)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared  sigma statistic p.value    df  logLik    AIC    BIC
##       <dbl>         <dbl>  <dbl>     <dbl>   <dbl> <dbl>   <dbl>  <dbl>  <dbl>
## 1     0.944         0.944 36459.    11299.       0     6 -47819. 95654. 95705.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Higher adjusted R^2 – better!

- Note: χ^2 can only be used for models on the same data
  - Same for AIC

Model accuracy

> Why is the UOL model better than the Singapore model?

- Ranking:
  1. Worldwide real estate model
  2. UOL model
  3. Singapore real estate model

Practice: group_by()

- This practice is to make sure you understand how to use mutate with leads and lags when there are multiple companies in the data
  - We'll almost always work with multiple companies!
- Do exercises 2 and 3 on today's R practice file:
  - R Practice
  - Shortlink: rmc.link/420r2

Dealing with noise

Noise

> Statistical noise is random error in the data

- Many sources of noise:
  - Other factors not included in
  - Error in measurement
    - Accounting measurement!
  - Unexpected events / shocks

> Noise is OK, but the more we remove, the better!

Removing noise: Singapore model

- Different companies may behave slightly differently
  - Control for this using a *Fixed Effect*
  - Note: ISIN uniquely identifies companies

```
forecast3.1 <-
  lm(revt_lead ~ revt + act + che + lct + dp + ebit + factor(isin),
     data=df_clean[df_clean$fic=="SGP",])
# n=7 to prevent outputting every fixed effect
print(tidy(forecast3.1), n=15)
```

```
## # A tibble: 27 x 5
##    term                        estimate std.error statistic   p.value
##    <chr>                          <dbl>     <dbl>     <dbl>     <dbl>
##  1 (Intercept)                   1.58     39.4        0.0401 0.968
##  2 revt                          0.392     0.0977     4.01   0.0000754
##  3 act                          -0.0538    0.0602    -0.894  0.372
##  4 che                           0.304     0.177      1.72   0.0869
##  5 lct                           0.392     0.0921     4.26   0.0000276
##  6 dp                            4.71      1.73       2.72   0.00687
##  7 ebit                         -0.851     0.327     -2.60   0.00974
##  8 factor(isin)SG1AA6000000 218.       76.5        2.85   0.00463
##  9 factor(isin)SG1AD8000002 -11.7      67.4       -0.174  0.862
## 10 factor(isin)SG1AE2000006   4.02     79.9        0.0503 0.960
## 11 factor(isin)SG1AG0000003 -13.6      61.1       -0.223  0.824
## 12 factor(isin)SG1BG1000000  -0.901    69.5       -0.0130 0.990
## 13 factor(isin)SG1BI9000008   7.76     64.3        0.121  0.904
## 14 factor(isin)SG1DE5000007 -10.8      61.1       -0.177  0.860
## 15 factor(isin)SG1EE1000009  -6.90     66.7       -0.103  0.918
## # ... with 12 more rows
```

Removing noise: Singapore model

```r
glance(forecast3.1)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic   p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>     <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.856         0.844  208.      69.4 1.15e-111    26 -2223. 4502. 4609.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```r
anova(forecast3, forecast3.1, test="Chisq")
```

```
## Analysis of Variance Table
##
## Model 1: revt_lead ~ revt + act + che + lct + dp + ebit
## Model 2: revt_lead ~ revt + act + che + lct + dp + ebit + factor(isin)
##   Res.Df      RSS Df Sum of Sq Pr(>Chi)
## 1    324 14331633
## 2    304 13215145 20   1116488   0.1765
```

This isn’t much different. Why? There is another source of noise within Singapore real estate companies

Another way to do fixed effects

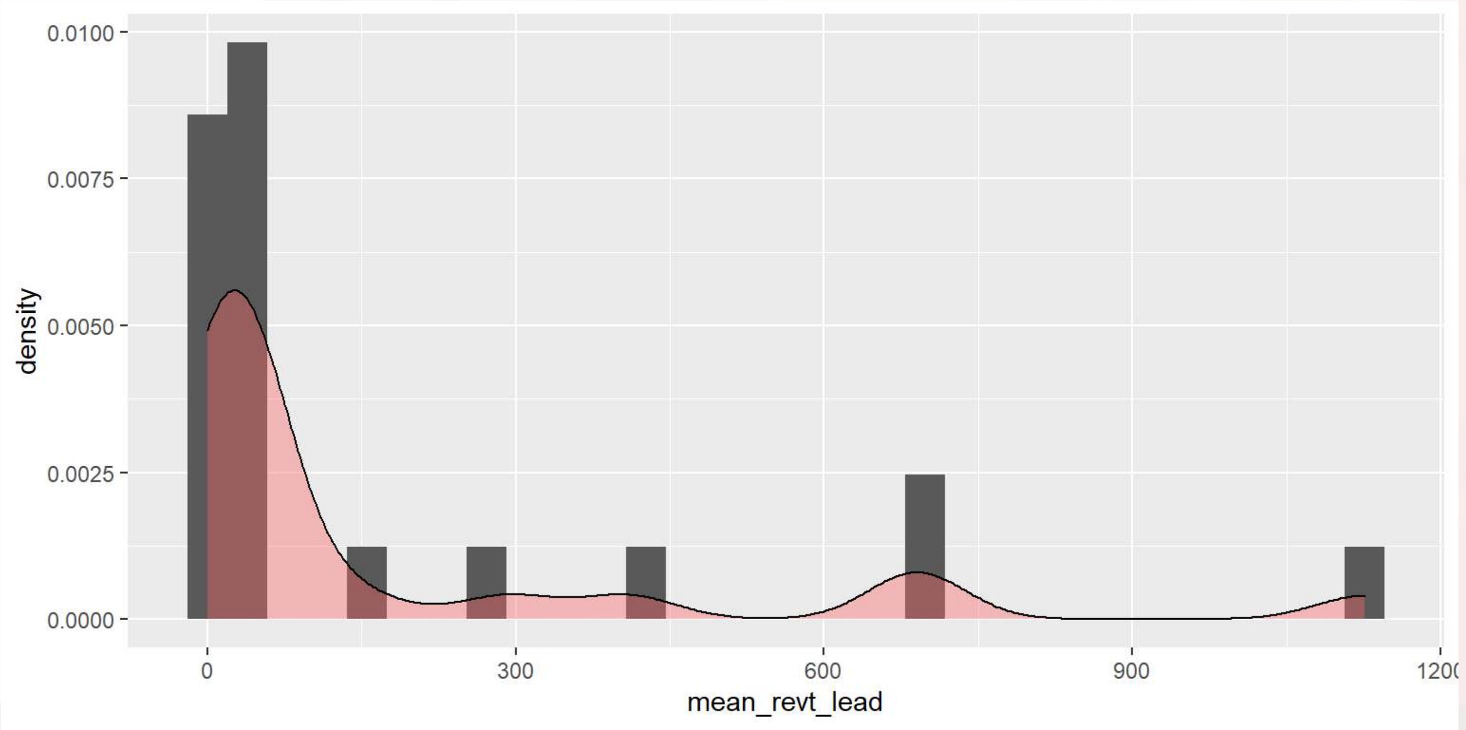
- The library `fixest` has `feols()`: fixed effects **OLS***
  - Better for complex models
  - Extremely efficient computationally

```r
library(fixest)
forecast3.2 <-
  feols(revt_lead ~ revt + act + che + lct + dp + ebit | isin,
        data=df_clean[df_clean$fic=="SGP",])
summary(forecast3.2)
```

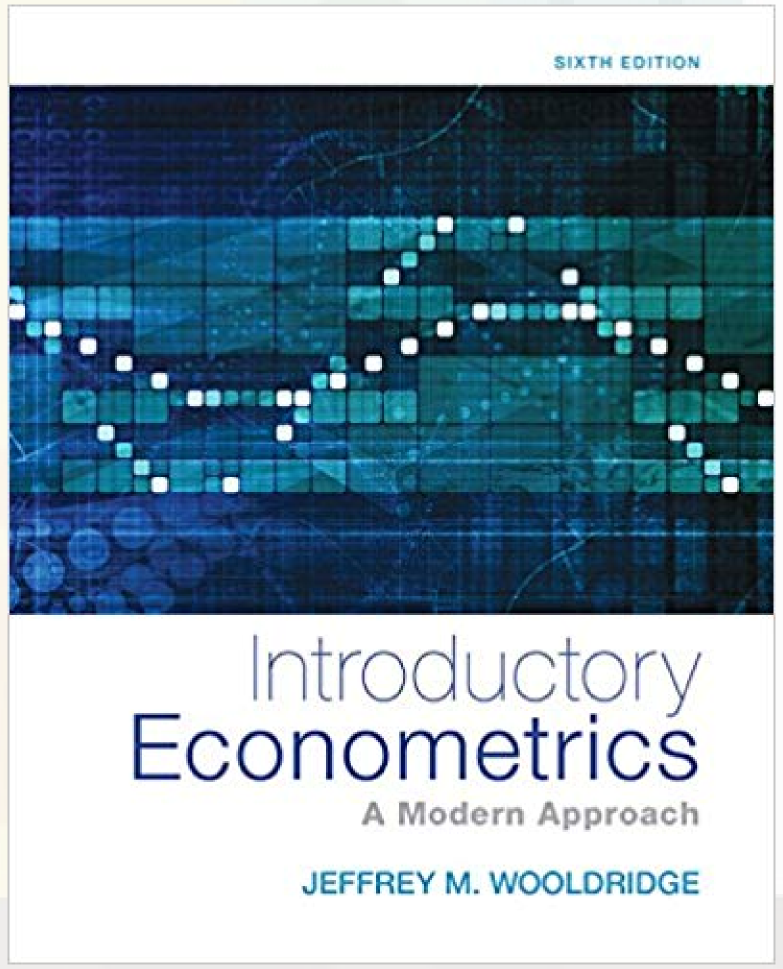
```
## OLS estimation, Dep. Var.: revt_lead
## Observations: 331
## Fixed-effects: isin: 21
## Standard-errors: Clustered (isin)
##        Estimate Std. Error   t value Pr(>|t|))
## revt  0.392002   0.188714  2.077200  0.050877 .
## act  -0.053816   0.154148 -0.349119  0.730649
## che   0.303696   0.302854  1.002800  0.327946
## lct   0.392086   0.238711  1.642500  0.116115
## dp    4.712800   2.630500  1.791600  0.088347 .
## ebit -0.850798   0.779077 -1.092100  0.287788
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## RMSE: 199.8      Adj. R2: 0.843506
##                Within R2: 0.780586
```

Why exactly would we use fixed effects?

- Fixed effects are used when the average of $\hat{y}$ varies by some group in our data
  - In our problem, the average revenue of each firm is different
- Fixed effects absorb this difference



- Further reading:
  - Introductory Econometrics by Jeffrey M. Wooldridge



What else can we do?

> What else could we do to improve our prediction model?

- Assuming: We have access to any data that is publicly available

End matter

For next week

- For next week:
  - 2 chapters on Datacamp
  - First assignment
    - Turn in on eLearn before class in 2 weeks
    - You can work on this in *pairs* or *individually*

Packages used for these slides

- `broom`
- `DT`
- `fixest`
- `knitr`
- `magrittr`
- `plotly`
- `revealjs`
- `tidyverse`

Custom code

```r
# Graph showing squared error (slide 4.6)
uolg <- uol[,c("at","revt")]
uolg$resid <- mod1$residuals
uolg$xleft <- ifelse(uolg$resid < 0,uolg$at,uolg$at - uolg$resid)
uolg$xright <- ifelse(uolg$resid < 0,uolg$at - uolg$resid, uol$at)
uolg$ytop <- ifelse(uolg$resid < 0,uolg$revt - uolg$resid,uol$revt)
uolg$ybottom <- ifelse(uolg$resid < 0,uolg$revt, uolg$revt - uolg$resid)
uolg$point <- TRUE

uolg2 <- uolg
uolg2$point <- FALSE
uolg2$at <- ifelse(uolg$resid < 0,uolg2$xright,uolg2$xleft)
uolg2$revt <- ifelse(uolg$resid < 0,uolg2$ytop,uolg2$ybottom)

uolg <- rbind(uolg, uolg2)

uolg %>% ggplot(aes(y=revt, x=at, group=point)) +
         geom_point(aes(shape=point)) +
         scale_shape_manual(values=c(NA,18)) +
         geom_smooth(method="lm", se=FALSE) +
         geom_errorbarh(aes(xmax=xright, xmin = xleft)) +
         geom_errorbar(aes(ymax=ytop, ymin = ybottom)) +
         theme(legend.position="none")
```

```r
# Chart of mean revt_lead for Singaporean firms (slide 12.6)
df_clean %>%                                         # Our data frame
  filter(fic=="SGP") %>%                             # Select only Singaporean firms
  group_by(isin) %>%                                 # Group by firm
  mutate(mean_revt_lead=mean(revt_lead, na.rm=T)) %>%  # Determine each firm's mean revenue (lead)
  slice(1) %>%                                       # Take only the first observation for each group
  ungroup() %>%                                      # Ungroup (we don't need groups any more)
  ggplot(aes(x=mean_revt_lead)) +                    # Initialize plot and select data
  geom_histogram(aes(y = ..density..)) +             # Plots the histogram as a density so that geom_density is visible
  geom_density(alpha=.4, fill="#FF6666")             # Plots smoothed density
```