

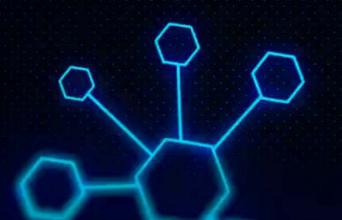
# ACCT 420: Logistic Regression for Bankruptcy

Dr. Richard M. Crowley

rcrowley@smu.edu.sg

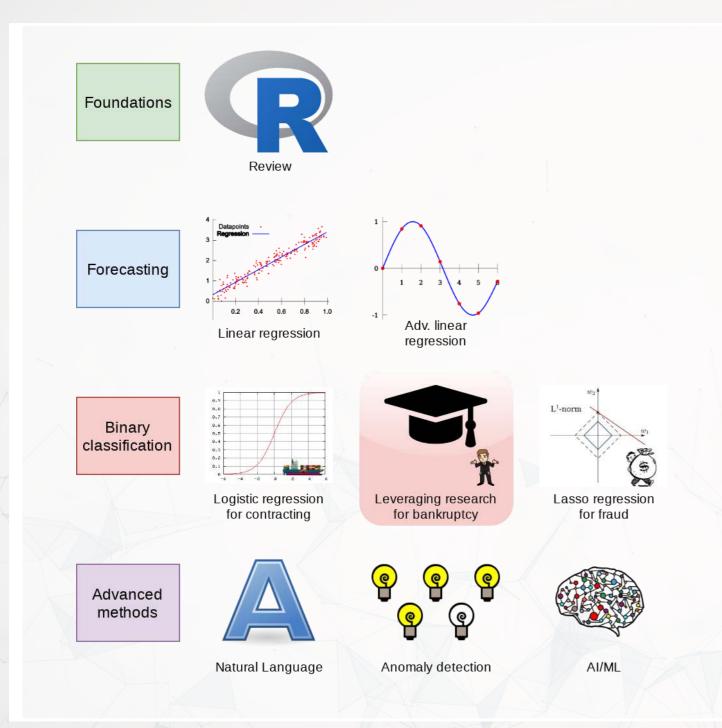
https://rmc.link/







## Learning objectives



- Theory:
  - Academic research
- Application:
  - Predicting bankruptcy over the next year for US manufacturing firms
    - Extend to credit downgrades
- Methodology:
  - Logistic regression
  - Models from academic research



# History of academic research in accounting

- Academic research in accounting, as it is today, began in the 1960s
  - What we call *Positive Accounting Theory* 
    - Positive theory: understanding how the world works
- Prior to the 1960s, the focus was on Prescriptive theory
  - How the world should work
- Accounting research builds on work from many fields:
  - Economics
  - Finance
  - Psychology
  - Econometrics
  - Computer science (more recently)

#### Types of academic research

- Theory
  - Pure economics proofs and simulation
- Experimental
  - Proper experimentation done on individuals
  - Can be psychology experiments or economic experiments
- Empirical/Archival
  - Data driven research
  - Based on the usage of historical data (i.e., archives)
  - Most likely to be easily co-optable by businesses and regulators



- Hedge funds
- Mutual funds
- Auditors
- Law firms
- Government entities like SG MAS and US SEC
- Exchanges like SGX



- The SMU library has access to almost all high quality accounting research
- Google Scholar is a great site to discover research past and present
- SSRN is the site to find cutting edge accounting and business research
  - List of top accounting papers on SSRN (by downloads)



#### First: Why care about bankruptcy?

- Read this article: rmc.link/420class5-1
  - "Carillion's liquidation reveals the dangers of shared sourcing"

Based on this article, why do we care about bankruptcy risk for other firms?

#### Where does the model come from?

- Altman 1968, Journal of Finance
- A seminal paper in Finance cited over 15,000 times by other academic papers

#### The Journal of FINANCE

Vol. XXIII

SEPTEMBER 1968

No. 4

#### FINANCIAL RATIOS, DISCRIMINANT ANALYSIS AND THE PREDICTION OF CORPORATE BANKRUPTCY

#### EDWARD I. ALTMAN\*

ACADEMICIANS SEEM to be moving toward the elimination of ratio analysis as an analytical technique in assessing the performance of the business enterprise. Theorists downgrade arbitrary rules of thumb, such as company ratio comparisons, widely used by practitioners. Since attacks on the relevance of ratio analysis emanate from many esteemed members of the scholarly world, does this mean that ratio analysis is limited to the world of "nuts and bolts"? Or, has the significance of such an approach been unattractively garbed and therefore unfairly handicapped? Can we bridge the gap, rather than sever the link, between traditional ratio "analysis" and the more rigorous statistical techniques which have become popular among academicians in recent years?

The purpose of this paper is to attempt an assessment of this issue—the quality of ratio analysis as an analytical technique. The prediction of corporate bankruptcy is used as an illustrative case.¹ Specifically, a set of financial and economic ratios will be investigated in a bankruptcy prediction context wherein a multiple discriminant statistical methodology is employed. The data used in the study are limited to manufacturing corporations.

A brief review of the development of traditional ratio analysis as a technique for investigating corporate performance is presented in section I. In section II the shortcomings of this approach are discussed and multiple discriminant analysis is introduced with the emphasis centering on its compatibility with ratio analysis in a bankruptcy prediction context. The discriminant model is developed in section III, where an initial sample of sixty-six firms is utilized to establish a function which best discriminates between companies in two mutually exclusive groups: bankrupt and non-bankrupt firms. Section IV reviews empirical results obtained from the initial sample and several secondary samples, the latter being selected to examine the reliability of the discriminant

- Assistant Professor of Finance, New York University. The author acknowledges the helpful suggestions and comments of Keith V. Smith, Edward F. Renshaw, Lawrence S. Ritter and the Journal's reviewer. The research was conducted while under a Regents Fellowship at the University of California Los Angeles.
- In this study the term bankruptcy will, except where otherwise noted, refer to those firms that are legally bankrupt and either placed in receivership or have been granted the right to reorganize under the provisions of the National Bankruptcy Act.

#### What is the model about?

- The model was developed to identify firms that are likely to go bankrupt out of a pool of firms
- Focuses on using ratio analysis to determine such firms

#### Model specification

$$Z = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.999x_5$$

- $x_1$ : Working capital to assets ratio
- $x_2$ : Retained earnings to assets ratio
- $x_3$ : EBIT to assets ratio
- $x_4$ : Market value of equity to book value of liabilities
- $x_5$ : Sales to total assets

This looks like a linear regression without a constant

#### How did the measure come to be?

- It actually isn't a linear regression
  - It is a clustering method called MDA (multiple discriminant analysis)
    - There are newer methods these days, such as SVM
- Used data from 1946 through 1965
  - 33 US manufacturing firms that went bankrupt, 33 that survived

More about this, from Altman himself in 2000: rmc.link/420class5-2

- Read the section "Variable Selection" starting on page 8
  - Skim through  $x_1, x_2, x_3, x_4$ , and  $x_5$

How would these assumptions stand today?





- Despite the model's simplicity and age, it is still in use
  - The simplicity of it plays a large part
- Frequently used by financial analysts

Recent news mentioning it



# Main question

Can we use bankruptcy models to predict supplier bankruptcies?

But first:

Does the Altman Z-score [still] pick up bankruptcy?



Is this a forecasting or forensics question?

- It has a time dimension like a forecasting question
- It has a feeling of a forensics question

#### The data

- Compustat provides data on bankruptcies, including the date a company went bankrupt
  - Bankruptcy information is included in the "footnote" items in Compustat
    - o If dlsrn == 2, then the firm went bankrupt
    - Bankruptcy date is dldte
- Most components of the Altman Z-Score model are in Compustat
  - But we'll pull market value from CRSP, since it is more complete
- All components of our later models are from Compustat as well
- Company credit rating data also from Compustat (Rankings)

# Bankruptcy in the US

- Chapter 7
  - The company ceases operating and liquidates
- Chapter 11
  - For firms intending to reorganize the company to "try to become profitable again" (US SEC)



## Common outcomes of bankruptcy

- 1. Cease operations entirely (liquidated)
  - In which case the assets are often sold off
- 2. Acquired by another company
- 3. Merge with another company
- 4. Successfully restructure and continue operating as the same firm
- 5. Restructure and operate as a new firm



# Calculating bankruptcy

```
# initial cleaning; 100338 is an outlier in the bonds distribution
df <- df \%>\% filter(at >= 1, revt >= 1, qvkey != 100338)
## Merge in stock value
df$date <- ymd(df$datadate)</pre>
df mve <- df mve %>%
 mutate(date = ymd(datadate),
         mve = csho * prcc f) %>%
 rename (qvkey=GVKEY)
df <- left join(df, df mve[,c("gvkey","date","mve")])</pre>
# Compute the bankruptcy indicator
df <- df %>%
 group by (gvkey) %>%
 arrange(datadate) %>%
 mutate(bankrupt = ifelse(row number() == n() & dlrsn == 2 &
                            !is.na(dlrsn), 1, 0),
         bankrupt lead = lead(bankrupt)) %>%
 ungroup() %>%
  filter(!is.na(bankrupt lead)) %>%
  mutate(bankrupt lead = factor(bankrupt lead, levels=c(0,1)))
```

- row\_number() gives the current row within the group; the first row is 1, next is 2, etc.
- n() gives the number of rows in the group

#### Calculating the Altman Z-Score

```
# Calculate the measures needed
df <- df %>%
  mutate(wcap at = wcap / at, # x1
         re at = re / at, \# x2
         ebit at = ebit / at, # x3
         mve lt = mve / lt, # x4
         revt at = revt / at) # x5
# cleanup
df <- df %>% mutate(across(where(is.numeric), ~replace(., !is.finite(.), NA)))
# Calculate the score
df \leftarrow df \%>\% mutate(Z = 1.2 * wcap at + 1.4 * re at + 3.3 * ebit at + 0.6 * mve lt + 0.999 * revt at)
# Calculate date info for merging
df$date <- ymd(df$datadate)</pre>
df$year <- year(df$date)</pre>
df$month <- month(df$date)</pre>
```

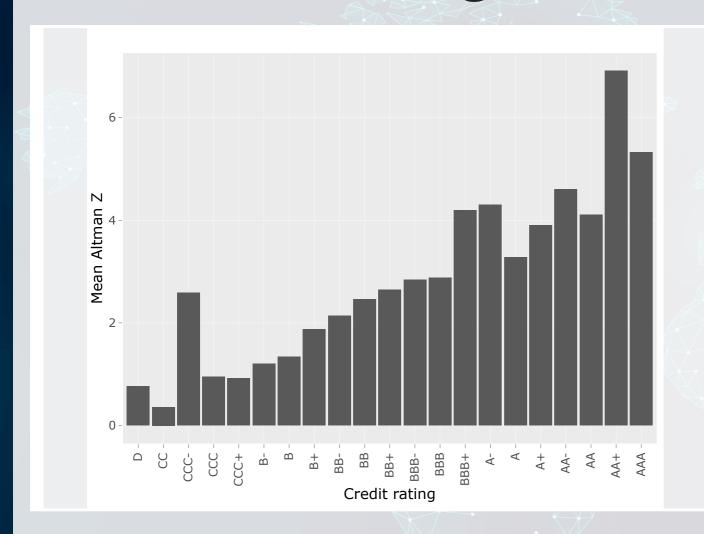
- Calculate  $x_1$  through  $x_5$
- Apply the model directly

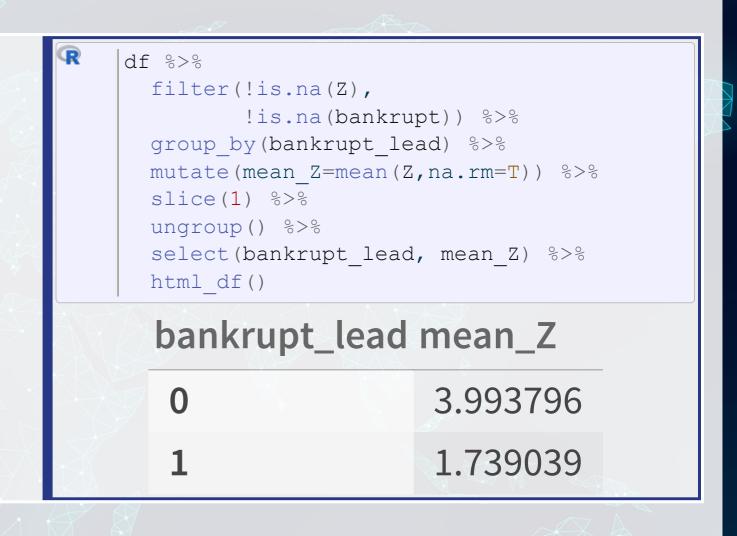
#### Build in credit ratings

We'll check our Z-score against credit rating as a simple validation

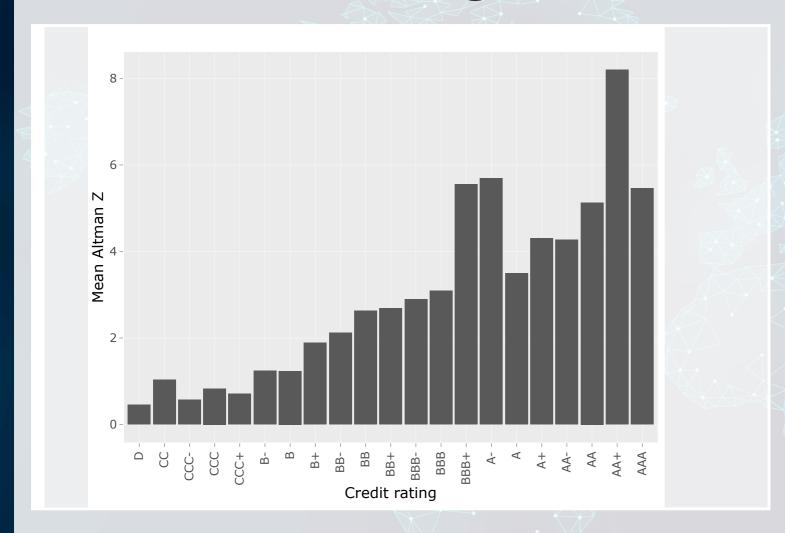
Credit rating is from Compustat – this data is no longer provided

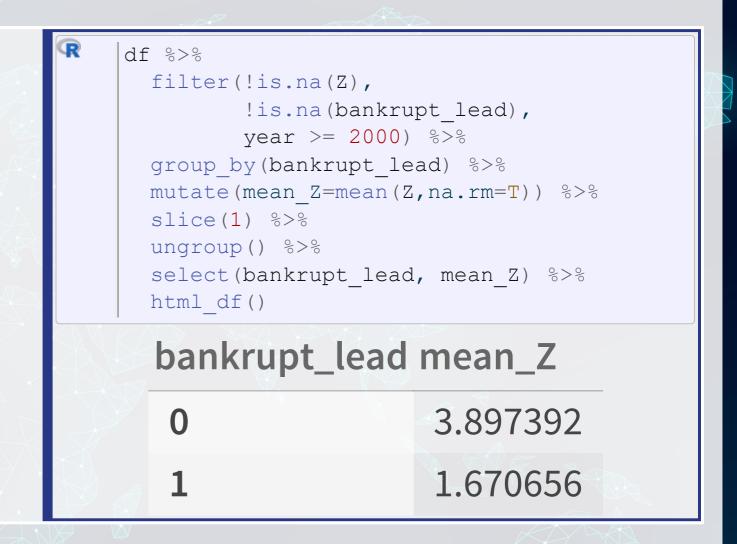
#### Z vs credit ratings, 1973-2017





## Z vs credit ratings, 2000-2017





#### Test it with a regression

```
fit Z <- glm(bankrupt lead ~ Z, data=df, family=binomial)</pre>
    summary(fit_Z)
Call:
glm(formula = bankrupt lead ~ Z, family = binomial, data = df)
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) -5.87769 0.11741 -50.060 < 2e-16 ***
           Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 1101.0 on 33372 degrees of freedom
Residual deviance: 1088.8 on 33371 degrees of freedom
```

## How good is the model though???

#### Examples:

Correct 92.6% of the time using Z < 1 as a cutoff

Only correctly captures 29 of 85 bankruptcies

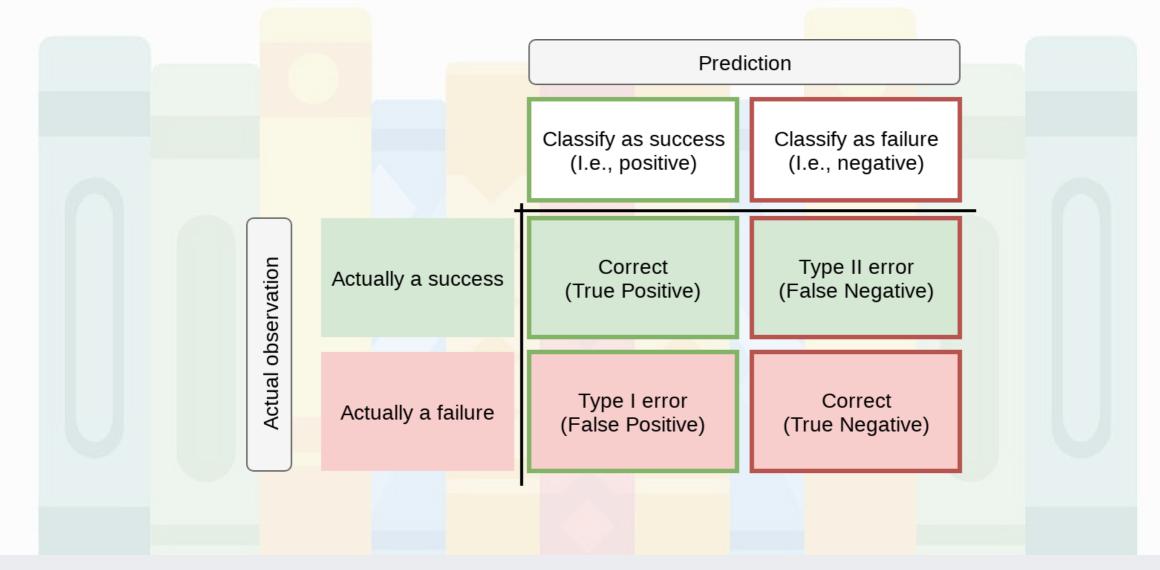
Correct 99.8% of the time if we say firms never go bankrupt...

Correctly captures 0 of 85 bankruptcies

```
Z < 1 Z >= 1
No bankruptcy 2654 30641
Bankruptcy 29 49
```



# Types of errors

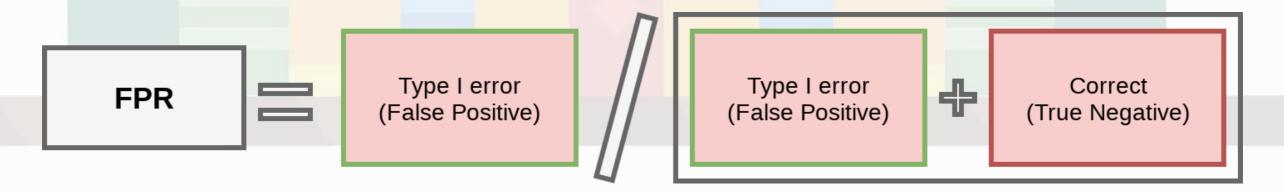


This type of chart (filled in) is called a *Confusion matrix* 

# Type I error (False positive)

We say that the company will go bankrupt, but they don't

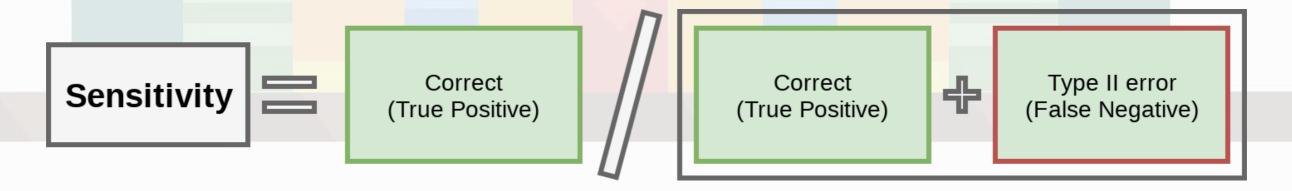
- A Type I error occurs any time we say something is true, yet it is false
- Quantifying type I errors in the data
  - False positive rate (FPR)
    - The percent of failures misclassified as successes
  - Specificity: 1 FPR
    - A.k.a. true negative rate (TNR)
    - The percent of failures properly classified



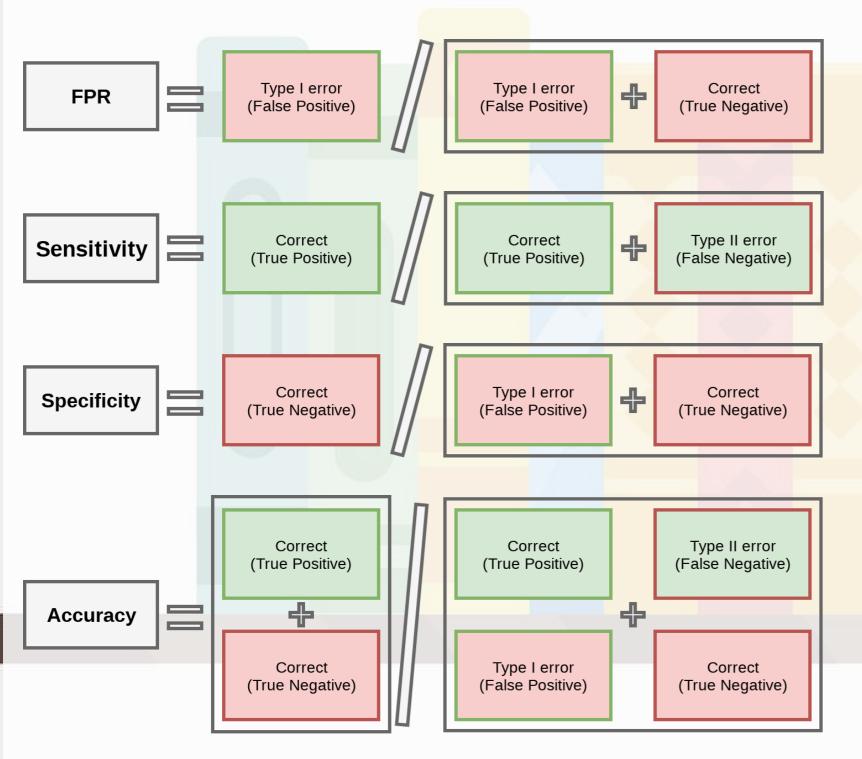
#### Type 2 error (False negative)

We say that the company will not go bankrupt, yet they do

- A Type II error occurs any time we say something is false, yet it is true
- Quantifying type I errors in the data
  - False negative rate (FNR): 1 Sensitivity
    - The percent of successes misclassified as failures
  - Sensitivity:
    - A.k.a. true positive rate (TPR)
    - The percent of successes properly classified



# Useful equations



#### A note on the equations

- Accuracy is very useful if you are predicting something that occurs reasonably frequently
  - Not too often, but not too rarely
- Sensitivity is very useful for rare events
- Specificity is very useful for frequent events
  - Or for events where misclassifying the null is very troublesome
    - Criminal trials
    - Medical diagnoses

Calculating any of these require the following

- 1. Predict percentages using predict(. , type="response")
- 2. Convert predictions to binary outcomes by specifying a desired cutoff
  - Anything below the cutoff is 0, anything above it is 1

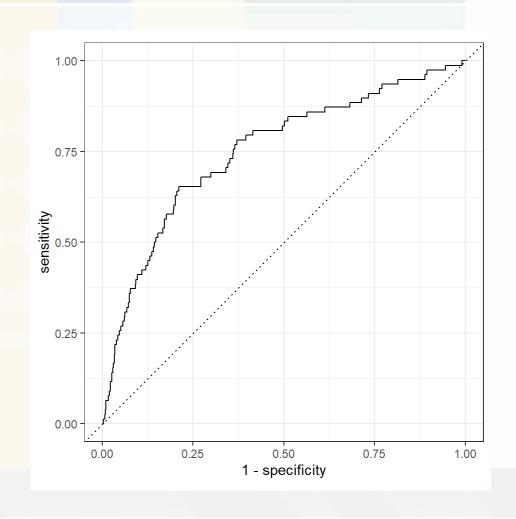
#### A more comprehensive approach

• Using yardstick we can plot out specificity and sensitivity across all possible cutoffs!

```
library(yardstick)
      df Z <- df %>% filter(!is.na(Z), !is.na(bankrupt lead))
     df Z$pred <- predict(fit Z, df Z, type="response")</pre>
     df Z %>% roc curve(bankrupt lead, pred, event level='second') %>% autoplot()
  1.00
  0.75
sensitivity
  0.25
  0.00
                             0.75
             0.25
                   1 - specificity
```

#### **ROC** curves

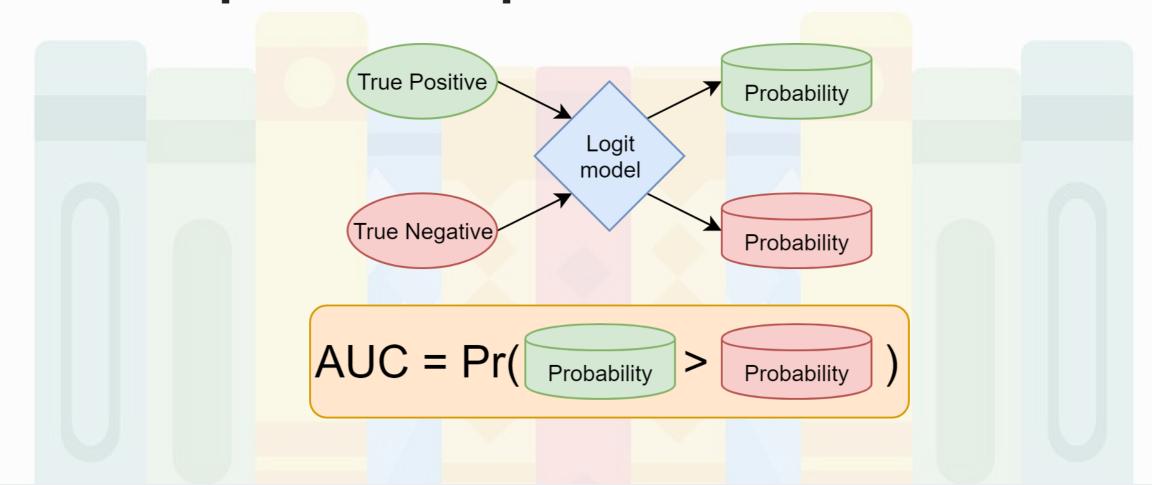
- The previous graph is called a ROC curve, or receiver operator characteristic curve
- The higher up and left the curve is, the better the logistic regression fits.
  - Neat properties:
    - The area under a perfect model is always 1
    - The area under random chance is always 0.5
      - This is the straight dashed line on the graph



#### **ROC AUC**

- The neat properties of the curve give rise to a useful statistic: ROC AUC
  - AUC = Area under the curve
- Ranges from 0 (perfectly incorrect) to 1 (perfectly correct)
- Above 0.6 is generally the minimum acceptable bound
  - 0.7 is preferred
  - 0.8 is very good
- yardstick can calculate this too

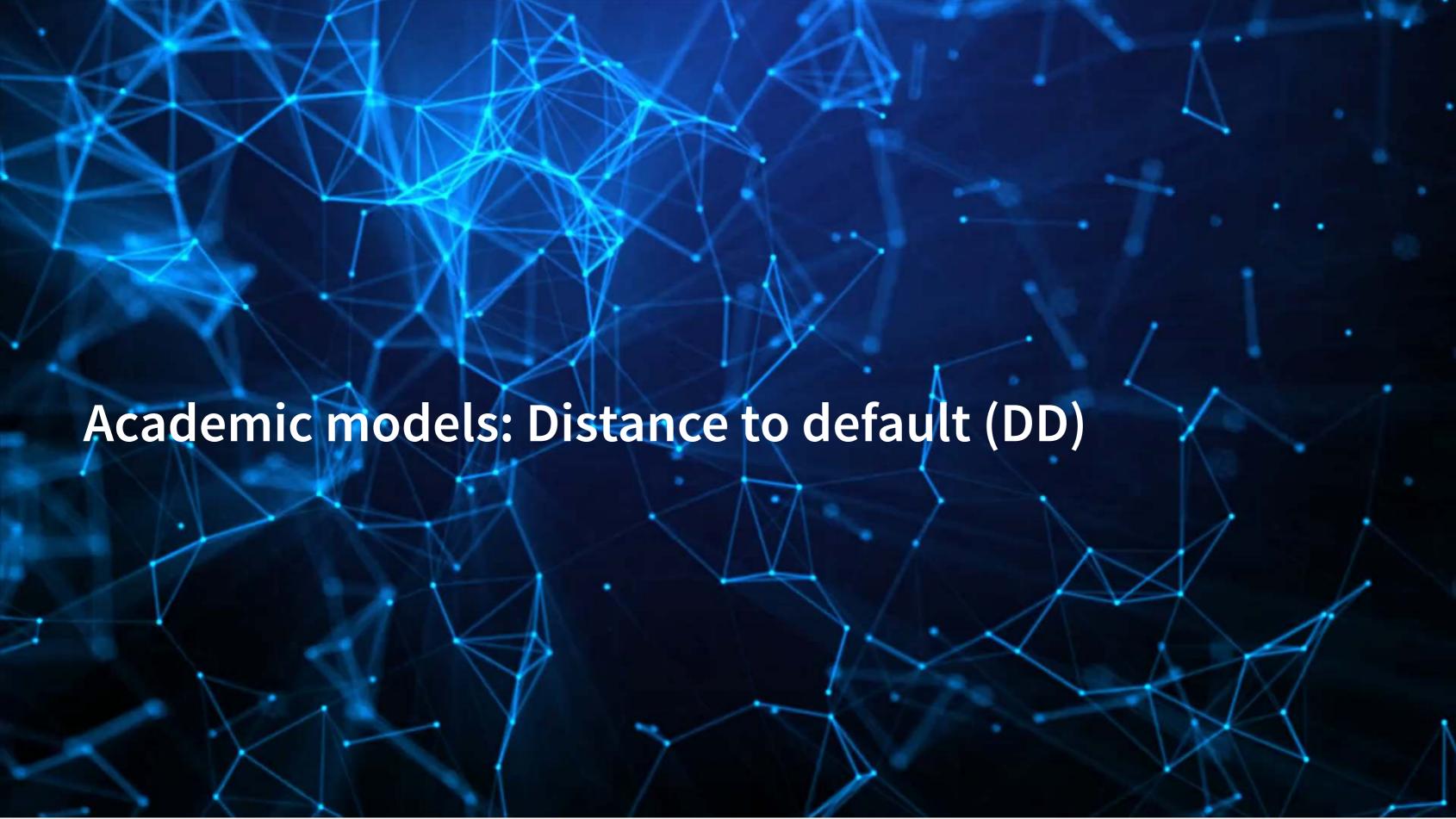
#### **ROC AUC simplest interpretation**



AUC is the probability that our model assigns a higher estimated probability to a randomly selected 1 than to a randomly selected 0.

#### R Practice ROC AUC

- Practice using these new functions with last week's Walmart data
  - 1. Model decreases in revenue using prior quarter YoY revenue growth
  - 2. Explore the model using predict()
  - 3. Calculate ROC AUC
  - 4. Plot a ROC curve
- Do all exercises in today's practice file
  - R Practice
  - Shortlink: rmc.link/420r5



#### Where does the model come from?

- Merton 1974, Journal of Finance
- Another seminal paper in finance, cited by over 12,000 other academic papers
- About Merton

#### ON THE PRICING OF CORPORATE DEBT: THE RISK STRUCTURE OF INTEREST RATES\*

#### ROBERT C. MERTON\*

#### I. INTRODUCTION

The value of a particular issue of corporate debt depends essentially on three items: (1) the required rate of return on riskless (in terms of default) debt (e.g., government bonds or very high grade corporate bonds); (2) the various provisions and restrictions contained in the indenture (e.g., maturity date, coupon rate, call terms, seniority in the event of default, sinking fund, etc.); (3) the probability that the firm will be unable to satisfy some or all of the indenture requirements (i.e., the probability of default).

While a number of theories and empirical studies has been published on the term structure of interest rates (item 1), there has been no systematic development of a theory for pricing bonds when there is a significant probability of default. The purpose of this paper is to present such a theory which might be called a theory of the risk structure of interest rates. The use of the term "risk" is restricted to the possible gains or losses to bondholders as a result of (unanticipated) changes in the probability of default and does not include the gains or losses inherent to all bonds caused by (unanticipated) changes in interest rates in general. Throughout most of the analysis, a given term structure is assumed and hence, the price differentials among bonds will be solely caused by differences in the probability of default.

In a seminal paper, Black and Scholes [1] present a complete general equilibrium theory of option pricing which is particularly attractive because the final formula is a function of "observable" variables. Therefore, the model is subject to direct empirical tests which they [2] performed with some success. Merton [5] clarified and extended the Black-Scholes model. While options are highly specialized and relatively unimportant financial instruments, both Black and Scholes [1] and Merton [5, 6] recognized that the same basic approach could be applied in developing a pricing theory for corporate liabilities in general.

In Section II of the paper, the basic equation for the pricing of financial instruments is developed along Black-Scholes lines. In Section III, the model is applied to the simplest form of corporate debt, the discount bond where no coupon payments are made, and a formula for computing the risk structure of interest rates is presented. In Section IV, comparative statics are used to develop graphs of the risk structure, and the question of whether the term premium is an adequate measure of the risk of a bond is answered. In Section V, the validity in the presence of bankruptcy of the famous Modigliani-Miller

Associate Professor of Finance, Massachusetts Institute of Technology. I thank J. Ingersoll
for doing the computer simulations and for general scientific assistance. Aid from the National
Science Foundation is gratefully acknowledged.

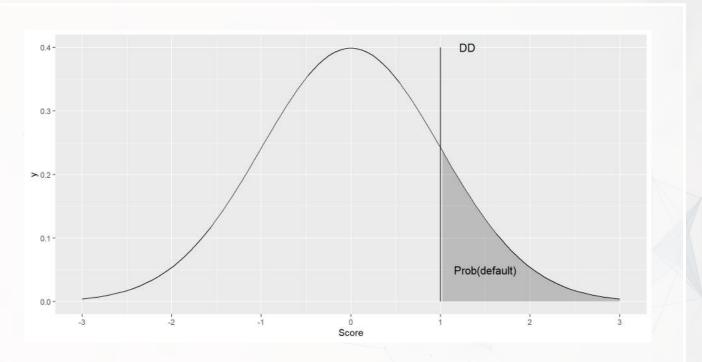


- The model itself comes from thinking of debt in an options pricing framework
- Uses the Black-Scholes model to price out a company
- Consider a company to be bankrupt when the company is not worth more than the the debt itself, in expectation

## Model specification

$$DD = rac{\log(V_A/D) + (r - rac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt(T-t)}$$

- $V_A$ : Value of assets
  - Market based
- D: Value of liabilities
  - From balance sheet
- r: The risk free rate
- $\sigma_A$ : Volatility of assets
  - Use daily stock return volatility, annualized
    - $\circ$  Annualized means multiply by  $\sqrt{253}$
- T-t: Time horizon



#### Who uses it?

- Moody's KMV is derived from the Merton model
  - Common platform for analyzing risk in financial services
  - More information

# MODYS



### Calculating DD in R

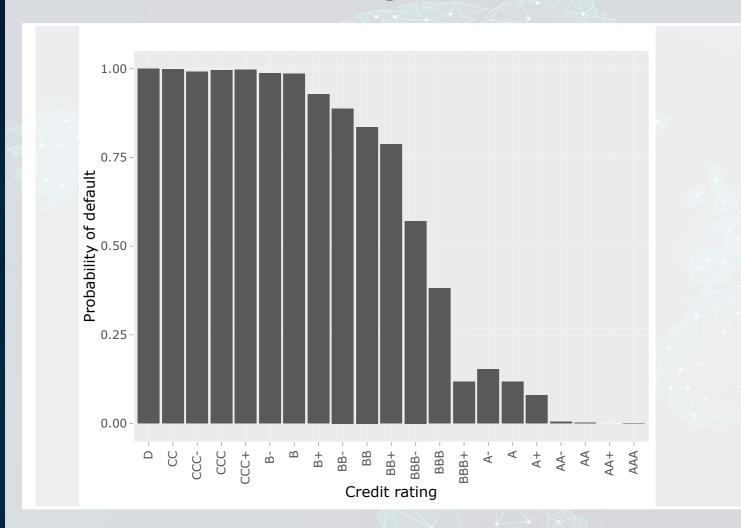
- First we need one more measure: the standard deviation of assets
  - This varies by time, and construction of it is subjective
  - We will use standard deviation over the last 5 years

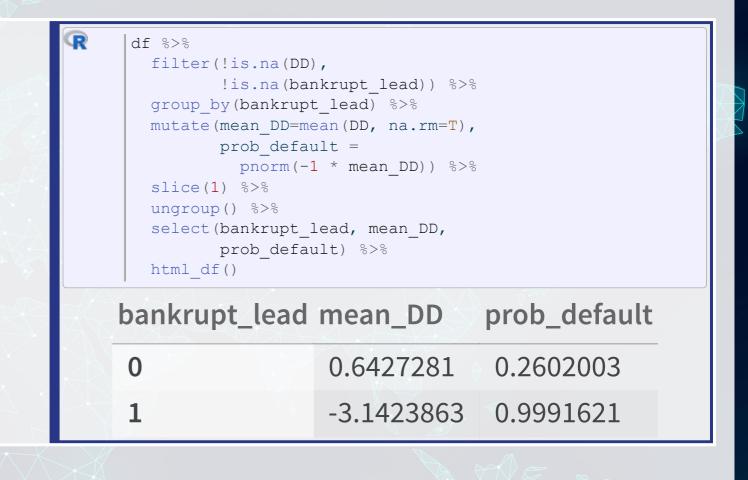
```
# df stock is an already prepped csv from CRSP data
df stock$date <- as.Date(df stock$date)</pre>
df <- left join(df, df stock[,c("gvkey", "date", "ret", "ret.sd")])</pre>
```

# Calculating DD in R

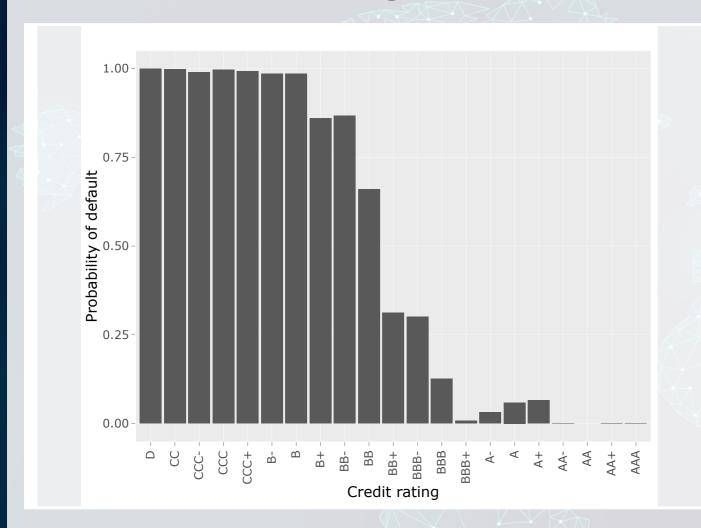
- Just apply the formula using mutate
- ullet  $\sqrt{253}$  is included because ret.sd is daily return standard deviation
  - There are ~253 trading days per year in the US

#### DD vs credit ratings, 1973-2017





#### DD vs credit ratings, 2000-2017



```
df %>%
  filter(!is.na(DD),
         !is.na(bankrupt_lead),
         year >= 2000) %>%
  group_by(bankrupt_lead) %>%
  mutate(mean DD=mean(DD, na.rm=T),
         prob default =
          pnorm(-1 * mean_DD)) %>%
  slice(1) %>%
  ungroup() %>%
  select(bankrupt_lead, mean_DD,
         prob default) %>%
  html_df()
bankrupt_lead mean_DD
                                 prob_default
                   0.8878013
                                  0.1873238
                   -4.4289487
                                  0.9999953
```

## Test it with a regression

```
fit DD <- glm(bankrupt lead ~ DD, data=df, family=binomial)
    summary(fit DD)
Call:
glm(formula = bankrupt lead ~ DD, family = binomial, data = df)
Coefficients:
          Estimate Std. Error z value Pr(>|z|)
-0.29783 0.03877 -7.682 1.57e-14 ***
DD
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 665.03 on 20455 degrees of freedom
Residual deviance: 608.65 on 20454 degrees of freedom
```

#### **ROC Curves**

```
df DD <- df %>% filter(!is.na(Z), !is.na(bankrupt lead))
     df_DD$pred <- predict(fit_DD, df_DD, type="response")</pre>
     df_DD %>% roc_curve(bankrupt_lead, pred, event_level='second') %>%
       autoplot()
 1.00 -
 0.75
              0.50
         0.25
           1 - specificity
    df_DD %>% roc_auc(bankrupt_lead, pred, event_level='second')
\# A tibble: 1 \times 3
  .metric .estimator .estimate
  <chr> <chr>
                           <dbl>
                           0.843
1 roc auc binary
```

#### **AUC** comparison

```
#AUC
auc_DD <- df_DD %>% roc_auc(bankrupt_lead, pred, event_level='second')
AUCs <- c(auc_Z$.estimate, auc_DD$.estimate)
names(AUCs) <- c("Z", "DD")
AUCs

Z DD
0.7498970 0.8434707
```

Distance to Default performs quite a bit better here!



#### A more practical application

- Companies don't only have problems when there is a bankruptcy
  - Credit downgrades can be just as bad

#### Why?

• Credit downgrades cause an increase in interest rates for debt, leading to potential liquidity issues.

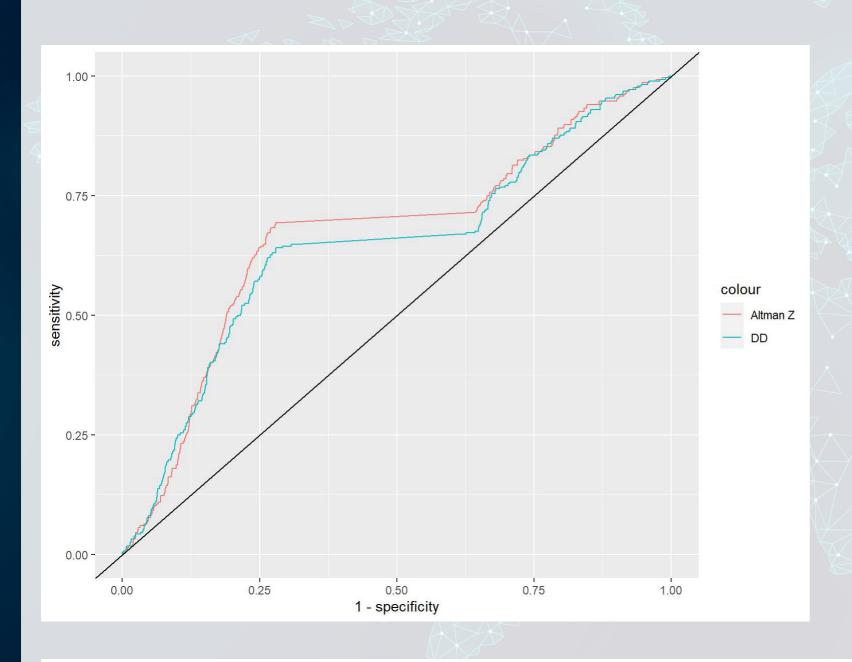
# **Predicting downgrades**

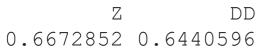
```
# calculate downgrade
df <- df %>%
 group by (gvkey) %>%
 arrange(date) %>%
 mutate (downgrade = factor (ifelse (lead (rating) < rating, 1, 0), levels=c(0, 1)),
         diff Z = Z - lag(Z),
         diff DD = DD - lag(DD)) %>%
 ungroup()
# training sample
train <- df %>% filter(year < 2014, !is.na(diff Z), !is.na(diff DD), !is.na(downgrade),
                       year > 1985)
test <- df %>% filter(year >= 2014, !is.na(diff Z), !is.na(diff DD), !is.na(downgrade))
# glms
fit Z2 <- glm(downgrade ~ diff Z, data=train, family=binomial)</pre>
fit DD2 <- glm(downgrade ~ diff DD, data=train, family=binomial)</pre>
```

# Predicting downgrades with Altman Z

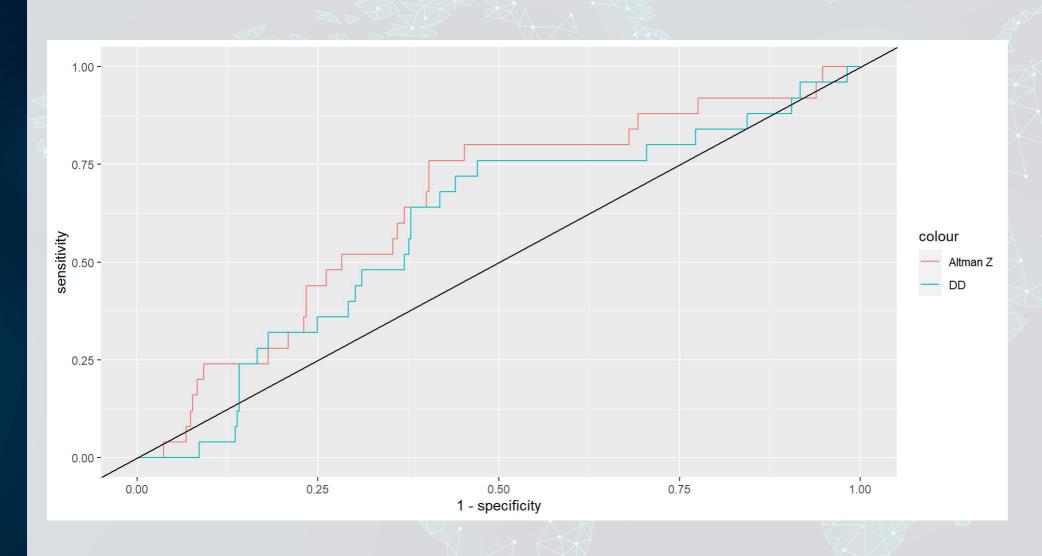
# Predicting downgrades with DD

### **ROC** Performance on this task





# Out of sample ROC performance



Z DD 0.6464 0.5904

# **Predicting bankruptcy**

What other data could we use to predict corporate bankruptcy as it relates to a company's supply chain?

- What is the reason that this event or data would be useful for prediction?
  - I.e., how does it fit into your mental model?
- A useful starting point from McKinsey
  - rmc.link/420class5-3
    - Section "B. Sourcing"

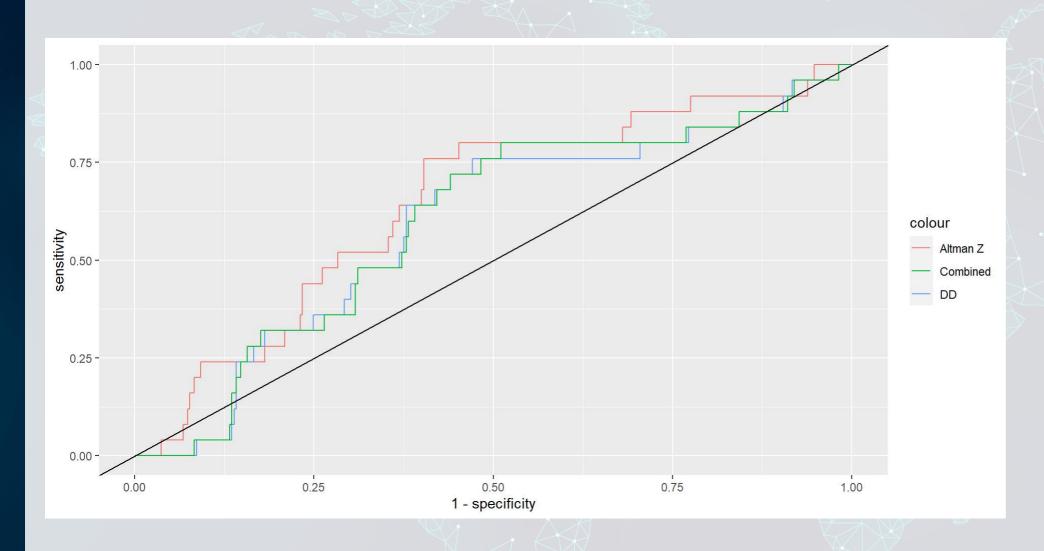


## Building a combined model

```
fit comb <- glm(downgrade ~ diff Z + diff DD, data=train, family=binomial)
    summary(fit comb)
Call:
glm(formula = downgrade ~ diff Z + diff DD, family = binomial,
   data = train)
Coefficients:
           Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.36899 0.06457 -36.689 < 2e-16 ***
diff Z
       0.02886 0.04289 0.673 0.501
diff DD -0.26787 0.04306 -6.220 4.97e-10 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
```

# Marginal effects

#### **ROC** Performance on this task



Z DD Combined 0.6464000 0.5904000 0.5959385



#### Wrap up

- For next week:
  - Second individual assignment
    - Finish by the end of the day next class session
    - Submit on eLearn
  - Datacamp
    - Practice a bit more to keep up to date
      - Using R more will make it more natural
  - Keep thinking about who you want to work with on the project
- Survey on the class session at this QR code:



# Packages used for these slides

- DT
- downlit
- kableExtra
- knitr
- margins
- plotly
- quarto
- revealjs
- tidyverse
- yardstick

#### **Custom code**

```
R
```

```
# Calculating a 253 day rolling standard deviation in R
crsp <- crsp %>%
  group_by(gvkey) %>%
  mutate(ret.sd = rollapply(data=ret, width=253, FUN=sd, align="right", fill=NA, na.rm=T)) %>%
  ungroup()
```